State of the art of generative design and topology optimization and potential research needs

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Abstract
Additive manufacturing allows us to build almost anything; traditional CAD however restricts us to known geometries and encourages the re-usage of previously designed objects, resulting in robust but nowhere near optimum designs. Generative design and topology optimization promise to close this chasm by introducing evolutionary algorithms and optimization on various target dimensions. The design is optimized using either 'gradient-based' programming techniques, for example the optimality criteria algorithm and the method of moving asymptotes, or 'non gradient-based' such as genetic algorithms SIMP and BESO. Topology optimization contributes in solving the basic engineering problem by finding the limited used material. The common bottlenecks of this technology, address different aspects of the structural design problem.

This paper gives an overview over the current principles and approaches of topology optimization. We argue that the identification of the evolutionary probing of the design boundaries is the key missing element of current technologies. Additionally, we discuss the key limitation, i.e. its sensitivity to the spatial placement of the involved components and the configuration of their supporting structure. A case study of a ski binding, is presented in order to support the theory and tie the academic text to a realistic application of topology optimization.

Keywords: topology optimization, product development, design, finite element analysis
1. Introduction

The ideal linkage between the additive manufacturing (AM) and the structural optimization (SO) is the key element in product development these days. On the one hand, models are produced by the addition of thousands of layers with the use of additive manufacturing (AM). That offers to designers a huge geometrical flexibility, with no additional cost, compared to traditional manufacturing. AM encompasses many technologies such as 3D printing, rapid prototyping and direct digital manufacturing (DDM). On the other hand, structural optimization reduces the material usage, shortens the design cycle and enhances the product quality. SO can be implemented according to size, shape, and topology (see Figure 1). Topology optimization is usually referred to as general shape optimization (Bendsøe, 1989). Most of the techniques optimize either the topology or both the size and the shape. There are only few examples that have tried to confront the problem in a holistic way (M. Zhou, Pagaldipti, Thomas, & Shyy, 2004).

![Illustration of a truss model and its different categories of structural optimization by: a) size, b) shape and c) topology (Bendsøe & Sigmund, 2003).](image)

The current state of the art of topology optimization (TO) is most oriented in the conceptual design phase. The general idea is to find the optimal material distribution of a structure with respect to its design and boundary constraints. However, the main challenge of TO is to provide a design parameterization that leads to a physically optimal design too (Sigmund & Petersson, 1998).

The first article about topology optimization was published in 1904 by the insightful Australian mechanical engineer Michell (1904). Michell’s article addressed the problem of least-volume topology of trusses with a single condition and a stress constraint. His contribution to topology optimization was the introduction of essential elements the so-called now, after a century, layout optimization, continuum-type optimally criteria, adjoin strain field and ground structure (Rozvany, 2009). After approximately 70 years it was Rozvany (1972) who extended Michell’s theory from trusses to beam systems and introduced the first general theoretical background of topology optimization termed ‘optimal layout theory’ (Rozvany, 1977). The scientific revolution in this field had begun and it has been mainly carried out the last 30 years with many interesting articles. There are three main approaches which deal with the topology optimization problem: element-based solution approaches (density, topological derivatives, level set, phase field, etc.), discrete approaches (evolutionary based algorithms) and combined approaches (Sigmund & Maute, 2013). The most known methods of topology optimization are: the solid isotropic material with penalization (SIMP) and the evolutionary structural optimization (ESO) or the bi-directional evolutionary structural optimization (BESO).
In the same direction, either gradient-based (optimality criteria algorithm, convex linearization, method of moving asymptotes, etc.) or non-gradient algorithms (genetic algorithms) were developed to support the theory of topology optimization.

Optimality criteria algorithm (OC) is the most fundamental gradient-based mathematical method. In this method, there is a proportional dependency between the design variables and the values of the objective function (Prager, 1968). The 99-line MATLAB by Sigmund (2001), which tackles the compliance problem for the Messerschmitt-Bölkow-Blohm (MBB) beam, is based on OC and nested analysis and design formulation (NAND). Convex linearization (CONLIN) is a linear mathematical programming method for structural optimization with mixed variables and respect to the problem’s characteristics. This method was introduced by Fleury and Braidbant (1986). Svanberg (1987) presented the method of moving asymptotes (MMA) which is a more aggressive version of CONLIN that is expanded by moving limits. The MMA creates an enormous sequence of improved feasible solutions of the examined problem. In addition to that, it can handle general non-linear problems and simultaneously take into account both constrains, design variables and characteristics of the structural optimization problem (cost, robustness, etc.). That was the foundation of the homogenization method (isotropic material) which was conducted the next year by Bendsøe and Kikuchi (1988) and a predecessor of the density-based approach of solid isotropic material with penalization (SIMP) (Bendsøe, 1989; M. Zhou & Rozvany, 1991).

The most notable non-gradient algorithms are the successive linear programming (SLP) and the successive quadratic programming (SQP). Both these methods transform the non-linear problem to a linear at a design point and optimize it within a limited region by movable boundary limits (Dantzig, 1963).

The aim of this paper is to give an overview over the different topology optimization approaches and practices. In addition, we run a case study of a ski binding using different practices of design optimization in order to implement the approaches and identify their needs. Of particular interest is the problem of the a priori fixed boundary and the real nearby limits to the potential designs and solutions.

2. Topology optimization (TO) and Finite Element Analysis (FEA)

Topology optimization is an iterative procedure adapted to the computer-added design (CAD). The main goal of this method is the best structural performance through the identification of the optimum material distribution inside the available volume of a structure with respect to its loads, boundary conditions and constraints. If TO is integrated into the traditional finite element analysis, the procedure can be divided to 8 steps as it is shown in Figure 2. This figure illustrates the geometry shift of a structure from its original geometry to topology geometry. In the beginning, FEA is implemented. It is possible to be used geometric modifications in order to simplify the initial problem. This stage is challenging to be computerized because it involves applying experience and judgement in a qualitative manner. However, the most crucial step at FEA is the definition of the problem statement and its equivalent mathematical model with all the required parameters (material properties, loads and restraints). The optimum results occur through the discretization (meshing) of the model and with a repetitive convergence method. The topology optimization method offers a new optimized design geometry with a notable mass reduction (or increment) which can be used as a new starting point for the FEA. Finally, the new FEA results validate or evaluate the success of the TO approach.
Figure 2: The geometry shift model of a cantilever beam with Abaqus based on the Kurowski FEA model (2017, pp. 10-11) and Simulia’s ATOM lifecycle.

3. The general topology optimization problem
The general mathematical solution of a continuous element-based optimization problem seeks the minimum (top down) or maximum (bottom up) value of a function \( f(x) \) and its related variable vector \( x = (x_1, \ldots, x_n) \in \mathbb{R}^n \) which generates it, with respect to possible conditions and constrains. According to Hassani and Hinton (1999, p. 3), the \( f \) can be called the objective or cost function and respectively the quantities \( x_i, i = 1, \ldots, n \) design variables and \( n \) the number of design variables. The design variables are depended due to equalities among the constrains, so it can be assumed that the real design space is a sub-space of \( \mathbb{R}^n \), where its dimension will be \( n \) minus the number of the independent equality constraints. Then the optimization problem can be expressed as:

\[
\begin{align*}
    f(x) & \quad \text{minimize this objective function} \\
    h_j(x) &= 0, \quad j = 1, \ldots, n_h \quad \text{equality constrains} \\
    g_k(x) &\leq 0, \quad k = 1, \ldots, n_g \quad \text{inequality constraints} \\
    x_i^l \leq x_i \leq x_i^u, \quad i = 1, \ldots, n \quad \text{design variables}
\end{align*}
\]

(1)

where
\( n_h \): number of equality constraints
\( n_g \): number of inequality constraints
\( n \): number of design variables
\( x_i^l \): lower bound of the design variable \( x_i \)
\( x_i^u \): upper bound of the design variable \( x_i \)

The term feasible domain can be used for the set of design variables which satisfy all the equality constrains and respectively infeasible domain the set of them which outrage at least one. Hence, there are either linear optimization problems, where both equality and inequality constraints are
linear functions of the design variables or non-linear optimization problems (most of the structural optimization problems), where at least one of the constraints is a non-linear function of the design variables (Hassani & Hinton, 1999, pp. 3-4).

4. **Topology Optimization approaches**

Topology optimization approaches can be categorized into element-based, discrete and combined, depending on the different algorithms they use.

4.1 **Element-based approaches**

The traditional topology optimization approaches are element-based. The general approach of these methods is the discretization of the problem domain in a number of finite elements whose solution is known or can be approximated. The definition of CAD geometry, by a number of solid elements and their connection points (nodes), is a prerequisite in FEM. These nodes have known degrees of freedom (loads, temperature, displacement, etc.). All the discrete solid elements of the model are used in their turn, in the definition of the mathematical interactions of node’s degrees of freedom and are combined to create the system’s equations. Finally, the solutions of these equations expose useful information about the system’s behavior (Thompson & Thompson, 2017, pp. 1-2).

As a consequence, topology optimization can extend the FEA-geometry of the model to the FEATO-geometry (combined FEA and TO geometry, see Figure 2). This iterative convergence method indicates either full material, partial material or lack of material to each solid element. The interpretation and verification of the TO’s results is a demanding procedure, especially in the case of combined size and shape optimization (Harzheim & Graf, 2005). The main challenge is that the building models have to be as close as their FEATO-geometry. If the interpretation of the results is not done properly from the designer, the whole optimization process will lose its significance (Cazacu & Grama, 2014).

The most notable element-based approaches are the density-based (gradient-based), the topological derivatives, the level set and phase filed approach.

At the density-based approaches, the basic topology optimization problem is tackled by discretizing the design domain $\Omega$ (allowable volume within the design can exist) using either solid elements or nodes. One of the most implemented and mathematically well-defined interpolation methodologies is the solid isotropic microstructure with penalization (SIMP). Other notable density-based methods are the rational approximation of material properties (RAMP), the optimal microstructure with penalization (OMP), the non-optimal microstructures (NOM) and the dual discrete programming (DDP) (Luo, Chen, Yang, Zhang, & Abdel-Malek, 2005; Rozvany, 2001; Sigmund & Maute, 2013).

Eschenauer et al. (1994) initiated the approach of topological derivatives known also with the name ‘bubble-method’. According to this approach, a microscopic hole (bubble with center $x$ and radius $\rho$) is introduced at point $x$ in or out of the design domain $\Omega$ in order to predict the influence (derivative) and trigger the creation of new holes. The bubble-method is a special case of homogenization, where the topological derivatives represent the limit of density going to 0 (void). These derivatives can indicate the ideal placing of a new hole or can be used either together with the level set approach or directly in element-based update schemes (Allaire, 1997; Burger, Hackl, & Ring, 2004; Eschenauer et al., 1994).
Level set models (Osher & Sethian, 1988) are characterized from their flexibility, dealing with demanding topological changes, due to implicit moving boundary (IMB) models (Jia, Beom, Wang, Lin, & Liu, 2011). These complex boundaries can form holes, split into multiple pieces, or merge with other boundaries to form a single surface. Hence, the adaptive design of the structure is carried out to solve the problem of structural topology optimization. At the traditional level set method (LSM), the boundary of structure is defined by the zero level (contour) of the level set function $\varphi(x)$. The zero level, in its turn, is derived by the objective function (such as energy of deformation, stress, etc.) and the optimal structure can be obtained through the movement and conjunction of its external boundary. The structure is defined by the domain $\Omega$, where the level set function takes positive values (Sigmund & Maute, 2013).

Phase field methods correspond to density approaches with explicit penalization and regularization. The initial approach was implemented by Bourdin and Chambolle in order to carry out perimeter constrains and represent the surface dynamics of phase transition phenomena, such as solid-liquid transitions (2003). This approach works directly on the density variables and is based on a continuous density field $\Omega$ which eliminates the need for penalization of interfaces between elements (Wallin & Ristinmaa, 2014).

### 4.2 Discrete approaches
As it was mentioned at section 3, the basic topology optimization problem uses discrete variables. Hence, it is reasonable to deal with it by formulating it instantly in discrete variables. However, this mathematical solution (sensitivity analysis) can be very challenging. In addition, this approach has some limitations with respect to size of problems and structures (Mathias Stolpe & Bendsøe, 2011). Nevertheless, there are some notable discrete approaches, such as the evolutionary structural optimization (ESO), additive evolutionary structural optimization (AESO) and the bidirectional evolutionary structural optimization (BESO), which have considerable efficiency.

### 4.3 Combined approaches
As it is mentioned at section 1, the most of the topology optimization methods use, as optimizing parameter, either only the topology of the elements/nodes or both the size and shape of the structure. There are not many approaches which try to confront the problem in a holistic way. Some notable combined topology optimization approaches are the extended finite element method (xFEM) (Van Miegroet & Duysinx, 2007) and the deformable simplicial complex (DSC) (Misztal & Bærentzen, 2012). On the one hand, the purpose of the xFEM was an introduction of a generalized and adaptive finite element scheme which could allow us to work with meshes that can represent smooth and accurate boundaries. On the other hand, DSC scheme combines nonparametric shape optimization approaches with the ability to introduce and remove holes.

### 5. Comparison of the different Topology Optimization approaches
At this section, is presented a comparison between the main topology optimization approaches with respect to their procedure (top down/bottom up), characteristics, strengths and weaknesses. The comparison is based on both review and research papers about topology optimization and is shown in Table 1.
Table 1. Comparison of the Topology Optimization Approaches

<table>
<thead>
<tr>
<th>Category</th>
<th>Element-based</th>
<th>Density-based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach</td>
<td>Procedure/Description</td>
<td>Strengths</td>
</tr>
<tr>
<td>Solid Isotropic Microstructures with Penalization (SIMP)</td>
<td>• Eulerian (fixed mesh) method • Discretization to solid isotropic elements • Remove material • Nested analysis and design approach (NAND) • Minimize the compliance subject to a volume constrain problem via an iterative converge method • ‘Soft-kill’ penalization method (white: void, gray: fractional material, black: material)</td>
<td>• Homogenization is not a prerequisite • Computational efficiency • Robustness • Adaptive to (almost) any design condition • Freely adjusted penalization • Conceptual simplicity (no higher mathematics required) • Available for all combinations of designs constrains</td>
</tr>
<tr>
<td>Rational Approximation of Material Properties (RAMP)</td>
<td>• Eulerian (fixed mesh) method • Based on SIMP • Nonzero sensitivity at zero density</td>
<td>• Convex</td>
</tr>
<tr>
<td>Optimal Microstructure with Penalization (OMP)</td>
<td>• Eulerian (fixed mesh) method • Based on SIMP • Discretization to optimal nonhomogeneous elements • ‘Hard-kill’ penalization method (white: void, black: material)</td>
<td>• More information about the isotropic-solid/empty/porous (ISEP) optimum</td>
</tr>
<tr>
<td>Non-Optimal Microstructures (NOM)</td>
<td>• Eulerian (fixed mesh) method • Based on SIMP • Discretization to nonoptimal nonhomogeneous elements • No penalization</td>
<td>• Available for all combinations of designs constrains • Less variables/element than OMP</td>
</tr>
<tr>
<td>Dual Discrete Programming (DDP)</td>
<td>• Eulerian (fixed mesh) method • Discretization to solid isotropic elements • Remove material</td>
<td>• Penalization is not necessary</td>
</tr>
<tr>
<td>Topological derivatives ('The Bubble-method')</td>
<td>• Lagrangian (boundary following mesh) method • Special case of homogenization • Remove material • Combine shape and topology optimization • Introduce microscopic hole in order to predict the influence (derivative) and trigger the creation of new holes</td>
<td>• Indirectly include filtering by mapping between nodal and element (or subelement) based on design variables.</td>
</tr>
<tr>
<td>Level set</td>
<td>• Eulerian (fixed mesh) and Hybrid methods • Operate with boundaries instead of local density variables. • Implicit moving boundary (IMB) models</td>
<td>• Flexibility in topological changes • Can be mesh-independent • Can find shape variations for robust design</td>
</tr>
<tr>
<td>Method</td>
<td>Description</td>
<td>Challenges</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-----------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| **Boundaries**                  | - Boundaries can form holes, split into multiple pieces, or merge with other boundaries to form a single surface  
- Boundary of structure = zero level (contour)  
- Modified density approach (uses shape derivatives for the development of the optimal topology)  
- Most use erset material and fixed meshes | - Formulate objectives and constraints on the interface and describe boundary conditions at the interface  
- Starting guess results  
- Regularization, control of the spatial gradients of the level set function, and size control of geometric features  
- Must combined with topological derivatives in 2d | |
6. Design optimization practices and examples

It is important to differentiate between a local optimum (a solution of a defined CAD model) and the optimal solution of a structural problem. A lot of design optimization practices have been developed the last years which try to combine both topology, shape and/or size optimization approaches in order to avoid the local optimum. Three main categories have been identified: a) predefined design space practice, b) maximum possible design space practice (with respect to boundary conditions) and c) integrated shape and topology optimization practice (IST). An overview of these practices is presented in Table 2.

Table 2. Overview of the design optimization practices

<table>
<thead>
<tr>
<th>Practice</th>
<th>Examples</th>
<th>Strengths</th>
<th>Weaknesses</th>
<th>Recom. papers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predefined design space</td>
<td>Upper carriage of a naval gun</td>
<td>Partially hold of the initial visual design</td>
<td>Restricted design space (fixed dimensions and boundary conditions)</td>
<td>(Wang &amp; Ma, 2014)</td>
</tr>
<tr>
<td>Maximum possible design space</td>
<td>Laser-remote-scanner</td>
<td>Larger design space (less restrictions on the algorithm)</td>
<td>Restricted design space (fixed dimensions and boundary conditions)</td>
<td>(Emmelmann, Kirchhoff, &amp; Beckmann, 2011)</td>
</tr>
<tr>
<td></td>
<td>Compressor bracket</td>
<td></td>
<td></td>
<td>(Chang &amp; Lee, 2008)</td>
</tr>
<tr>
<td></td>
<td>Trailer chassis</td>
<td></td>
<td></td>
<td>(Ma, Wang, Kikuchi, Pierre, &amp; Raju, 2006)</td>
</tr>
<tr>
<td></td>
<td>Hanger</td>
<td></td>
<td></td>
<td>(McKee &amp; Porter, 2017)</td>
</tr>
<tr>
<td>Integrated shape-and topology</td>
<td>Automotive Design and</td>
<td>Optimization of boundary conditions</td>
<td>Computationally costly and time consuming</td>
<td></td>
</tr>
<tr>
<td>optimization practice</td>
<td>Manufacturing</td>
<td></td>
<td></td>
<td>(Fiedler, Rolfe, &amp; De Souza, 2017)</td>
</tr>
</tbody>
</table>

7. Case study of a ski binding

In order to present the limitations of the topology optimization approaches and design practices, an example of a minimum compliance design of a ski binding will be presented. The optimization problem is restricted due to time and computational limitations. We assume that the applied forces are given and the ski binding is fixed to the ground with four screws. Hence, the topology optimization of the structure is conducted in conjunction with the optimization of the positions of the screws. In this case, the model was built in Abaqus CAE 2017 and the optimization was conducted using the optimization software Tosca Structure, which is based on SIMP topology optimization approach. First the three main practices were tested in our case and finally a new practice is recommended based on the identified limitations of the existing practices.

7.1 Topology optimization of a ski binding

In Figure 3, both the predefined design space and maximum possible design space practices of the ski binding are presented.
As it is shown in Figure 3, the maximum possible design space practice resulted to an optimum with a larger design space.

In Figure 4, is illustrated the mathematical model in the IST practice which consists of the design envelope, a set of screws and a contact set with a ski. As dynamic design parameters are used the distance $d$ between the pairs of the screws and the thickness $t$ of the support structure under the yellow area.

Two different studies are executed in order to detect the optimal solution. In the first study, the chosen thickness of the support structure is $t = 2\,mm$, while the distance $d$ between the pairs of screws are decreased by $5\,mm$ in each iteration in a range of $20.5 \to 35.5\,mm$ (see Figure 4). In the second study, the same screw-hole patterns are re-tested but now with thickness of the support structure, $t = 5\,mm$ (see Figure 5).

Comparing the results of the three practices, TO model 1 and TO model 6 from the IST practice are the solutions with the lowest strain energy (i.e. the stiffest result) and thus the local optimum in study one and two respectively.
7.2 Evaluation of the applied practices
It is clear that these practices led to a local optimum and not to the best optimized solution of the ski binding. It is crucial to understand that the optimum solution to a defined setup might not be the ideal solution to the problem. In other words, the optimum material distribution is influenced by the initial boundary conditions defined by the engineer. Therefore, it is not possible to find the real optimum of a structure, if its outer boundary conditions are not optimal and predefined. Then it is necessary to find first the optimum input (boundary conditions) for the topology optimization, and second the optimum material distribution.

In this case, the identification of the real optimum has been carried out by comparing the different optimized design models in the conducted practices. However, this methodology has several limitations such as the requirement of a huge amount of time and computer capacity due to the analysis of big data (different sizes and placements of screws, variation of the support structure for the loads and boundary conditions, etc.). This method also implies the need of all the setups to be defined by an engineer, making the final design more vulnerable to human error and his/her previous experience.

7.3 Suggestions about a new practice
The success of a topology optimization approach could be achieved through the identification of the evolutionary probing of the design boundaries. Hence, the topology optimization problem could be divided in two sub-problems (levels); the optimization of the outer boundary conditions, and the optimization for the inner optimum. The CAD-geometry of the structure could be replaced by a black box with the allowable design envelope (level 1). A topology optimization algorithm, based on NAND formulation, could be used for calculating the optimum adaptive (moving) boundaries of the structure with respect to outer design parameters (i.e. length, width, height, holes, etc.), constrains, loads and contact sets. The optimum boundary conditions could be used in their turn, as a starting point for a traditional topology optimization of the structure’s interior (level 2). The geometry shift model and the principle flow chart of this approach are presented in Figures 5 & 6 respectively.

Figure 6: The geometry shift model of a cantilever beam based on the two-level topology optimization approach with Abaqus 2017
A comparison between the traditional (Figure 2) and the two-level (Figure 6) geometry shift shows that both CAD and FEA geometries have been replaced by a more ‘generative design based’ optimization approach. This can result in optimum and high applicable structural designs with minimized human error and reduced number of convergence iterations.

Figure 7: Principle flow chart for the two-level topology optimization approach

8. Conclusion and future research potentials
In this paper, in the first section, was presented the general topology optimization problem and the most implemented topology optimization approaches. The most used and commercial applied method is the SIMP. ESO is also a promising method with many potentials, but it is still missing the mathematical background for multiple constraints and loads. However, all the approaches have their advantages and limitations. Both SIMP and ESO are dependent on the design parameters (CAD), mesh and boundary conditions of the structures.

In the second section, some design optimization practices were used in the case of a ski binding. Through the applied optimization practices and their results we agreed on the following limitations:

- CAD is a limited design methodology due to its design parameters restrictions to known geometries.
- CAD encourages the re-usage of previously designed objects resulting in robust but nowhere near optimum designs.
- The main key limitation of the topology optimization is its sensitivity to the spatial placement of the involved components and the configuration of their supporting structure. For example, the local optimum of the ski binding will be completely different if we use three screws instead of four.
- Many topology optimization approaches are still dependent on starting guesses.
- All the existing topology optimization approaches and practices are time consuming and demand huge computational effort when they try to tackle big 3D construction models.
It is clear that there is a need of new practices which could overcome these limitations. Suggestions about a new practice were presented. The main goal of this approach is to implement a two level optimization, first the outer bounder conditions optimum and as a consequence the inner optimum which is based on the first one. A geometry shift model and a principle flow chart of this approach were presented. Further research, validation of the applicability of this practice and the development of its mathematical formulation are needed to be done.

References


