1. Introduction
The collaborative research centre SFB805 at the Technical University of Darmstadt aims at reducing uncertainty in all processes of the product lifecycle. This includes the product development process and the ways in which designers obtain their solutions. Therefore, a holistic approach to controlling uncertainty must consider also the development phase and uncertainty related to designing itself (influence of experience, motivation, knowledge, etc). Robust design requires methods that consider variety in design parameters, and methods to consider variety during the design phase. Integrated product and process development according to [Birkhofer et al. 2012] illustrates the relations between the design phase and real processes, such as production and product use. While product development should anticipate the processes of the product lifecycle, decisions made during development also influence the product lifecycle. Variety in the design phase therefore also influences the real product and its processes.

![Figure 1. Integrated product and process development according to [Birkhofer et al. 2012]](image.png)

In literature, it is mostly accepted that Robust Design should be implemented as early as possible in the development process to maximize its impact [Andersson 1996]. [Ebro et al. 2012] introduce a method to proof design mobility, aiming at correctly constrained systems as early as possible to avoid functional loss and later iteration loops. To assess a design's movability the Kutzbach Grübler criterion is used. It is easy to apply, widely known and results can be obtained quickly. There are also some disadvantages that have to be taken into account:

- Knowledge of the kinematic type needs to be substantial to obtain the correct result.

Keywords: robust design, uncertainty, kinematic design
• The result is the actual number of independent degrees of freedom of the system; it contains no information on which degrees of freedom are available and why.
• For further development or deeper insights, an additional analysis of the system has to be conducted.

But there are alternatives. [Roth 1982] introduced a matrix-based analysis of technical systems that can be applied in robust design tasks. There exists in the robust design community some approaches based on this principle. [Söderberg et al. 2006] uses matrix-based location schemes to find optimum positions for sheet metal connections in the automotive industry. Eifler [2014] uses a matrix-based product model representation to consider variety in uncertain geometrical properties. The elemental interfaces approach [Freund et al. 2015, Freund et al. 2015] also contributes to this field. These approaches provide an alternative way to assess system mobility, based on the same matrix representation of products.

Derived from this, the research questions motivating this paper are:

Are matrix-based mobility analysis approaches an alternative to the Kutzbach Grübler criterion actually applied in robust design tasks? Do they work well? What are the advantages of the approaches and the possible shortfalls?

To answer these questions, mobility in robust design tasks is analysed theoretically and practically in Section 2 through an overview of existing research.

Section 3 shows the alternative analysis, introducing the elementary interface approach for kinematic analysis cases. The elemental interface approach is applied to the example in Section 2 to make the approach comparable to the Kutzbach Grübler Criterion.

Section 4 compares the outcomes of both approaches.

2. The Kutzbach Grübler criterion in robust design tasks

The Kutzbach Grübler criterion is one of the earliest measures that can be applied to analyse a system for robustness [Ebro et al. 2012]. During product development, analysis and synthesis steps alternate. Particularly in early phases, where a lot of time is spent synthesising new solution variants, it is imperative that all solutions provide the intended functions. Due to loss of overview of complex systems, a mechanism can easily provide a degree of freedom that does not match the intended movability, leading to the worst case scenario in cost-intensive redesigns. One way to prevent these scenarios is the Kutzbach Grübler criterion. It can be applied even to a simple concept sketch and allows comparison of the actual movability of a system with the intended movability. This results in a deeper understanding of the contribution of particular parts to system functionality and helps designers to check their solutions early on.

2.1 How it works

Generally, the Kutzbach Grübler criterion allows determination of a system's degrees of freedom. The corresponding mathematical statement is:

$$M(3d) = 6(n - 1) - \sum U - \sum F_{id} ; M(2d, \text{spherical}) = 3(n - 1) - \sum U - \sum F_{id}$$

(1)

where $M$ is the resulting movability, $n$ the amount of linked bodies, $U$ the amount of constraints and $F_{id}$ the number of identical freedoms. The criterion also differentiates between 3d and a 2d (plane and spherical).

The obtained movability indicates whether a system is under-constrained ($M > 0$), fully constrained ($M = 0$), or over-constrained ($M < 0$). If a system is under-constrained it is able to move, unless $M$ additional forces, movements, or constraints are added.

The major shortfall of the Kutzbach Grübler equation is that it is not universally applicable without knowledge of the system. The number of identical freedoms is hard to identify in complex systems and deriving all equations of motion for the whole system is time consuming. Additionally, information on whether a system is 3d kinematic or spherical cannot always be derived, although it has a huge impact on the result of the criterion.
2.2 Example case study

The Robust Design group of DTU, actually chairing the Robust Design SIG of the Design Society, provides a robust design workshop that investigates a mass consumer product, a glue gun [Eifler et al. 2015]. This example is also used in this paper to portray the results and render them comparable.

![Image of a glue gun and its mechanism diagram.](image)

The glue gun mainly consists of a heating element and a mechanism. The heating element melts the glue and disperses it through the nozzle. The trigger is used to interact with the user. An applied rotational movement of the trigger causes the mechanism to transform the rotation into a translation of the glue stick. The mechanism consists of 5 bodies. Body 1 is the trigger, body 2 and body 3 are bars, and body 4 is the glue stick carriage. Body 5 is the housing. Most of the links between the elements of the mechanism are rotational joints (1-4); link 5 is a prismatic joint. All of the links allow for 1 degree of freedom, which leads to 2 constraints each in the case of a plane system.

\[
M(2d) = 3(n - 1) - \sum U - \sum F_{id}; M = 3(5 - 1) - 5 \cdot 2 = 2
\]

(2)

Applying the Kutzbach Grübler Criterion to the glue gun system results in a mobility M of 2 ([Eifler et al. 2015]). This means that the system is under-constrained and has 2 possibilities of movement. In the case of the glue gun, applied rotation of the trigger (can be seen as an additional constraint) leads to a system with mobility of 1. This matches the expected mobility of the mechanism in the glue gun utilization processes. The rotation of the trigger causes a translation of the glue stick and the intended function can be obtained.

![Body link diagram of the glue gun mechanism. It consists of 4 rotational joints and 1 prismatic joint.](image)
3. The elemental interfaces approach applied in kinematic design

The elemental interfaces approach was introduced in [Freund et al. 2015, Freund et al. 2015]. It is based mainly on matrix-based product design by [Roth 1982]. The approach was introduced to assess design clarity, but there is potential to use the same technique to assess design kinematics. As mentioned previously, the Kutzbach Grübler criterion has some disadvantages that may be circumvented using a matrix-based approach directly.

3.1 How it works

3.1.1 Model

The underlying product model of the elemental interfaces approach is the Contact Channel Model (C&CM), originally by [Matthiesen 2002], (see also [Albers et al. 2006]). A structure can be seen as a combination of working bodies that interact with each other through working surfaces. A working surface pair builds an interface between working bodies. In [Freund et al. 2015], the model was enlarged to include categorised interface symbols to represent geometrical information directly in the C&CM (see also [Frei 2002]).

![Figure 4. Enlarged contact and channel model for a simple shaft to shaft connection, by [Freund et al. 2015]](image)

Figure 4 is the C&CM representation of a shaft to shaft connection. The two shafts are considered as working bodies and their relations are represented through three working surface pairs. Two of them are planar and one is cylindrical.

3.1.2 Method

Based on the product model, the interactions between the working bodies can be written in a binary coded matrix that considers every elemental interface and possible motion of each interface (Figure 5, Table 1).

Roth [1982] introduces two kinds of matrices to describe technical systems, both consisting of 12 values: 6 translations (3 in positive and 3 in negative axis direction) and 6 rotations (3 around positive and 3 around negative axis directions). The Contact Matrix only considers contacts between working bodies, not forces. To consider closing effects, such as frictional lock and forces, the connection type matrix is used. Using both, Roth derives a digitalised Connection type matrix, where a value of 1 means a lock of particular direction and 0 means freedom of the particular direction. The matrix representations of the elemental interfaces have to be combined to obtain information about the whole system. [Roth 1982] introduces Boolean operation rules that differentiate between serial (disjunct) and parallel (conjunct) joints.
Based on a differentiation between geometry of working surface pairs and intended functions to be realised through a particular working surface pair, [Freund et al. 2015] introduced a catalogue that allows direct derivation of the corresponding matrix representations and additional information about relevant design parameters (Figure 6).

![Figure 5. Representing relative system mobilities in matrices. Every direction is considered, with a value for positive mobility and a value for negative mobility [Roth 1982]](image)

![Figure 6. Catalogue of elemental interfaces, as in [Freund et al. 2015]. The example shows a plane working surface pair and the corresponding closing, closing type and digitalised closing matrix for physical effects and closing orientation](image)

### 3.1.3 Potentials for kinematic design

Design for Clarity is an approach that currently occurs in the embodiment design phase. Its strengths are the detailed analysis of the interactions between components and a deeper understanding of the system. This leads to correctly constrained systems and decisions can be made based on facts rather than feelings. While embodiment design contains a lot of information about a system, conceptual design often contains nothing more than a sketch to describe a system. Nevertheless, the intended function and the intended mobility are known from above the working principle level. The solutions may not include justified information about detailed geometries, but the general principle, some of the components and their interactions already exist. This means that the elemental interfaces approach to investigating the clarity of a system can also be applied to early phases. It is just the focus that slightly changes within product development; the method is always the same. In early phases, the aim of the method is to obtain information to assess functionality of derived concepts without consideration of exact and detailed geometry, for example, the correct number and type of joints in the glue gun concept, no matter how they are realised. Later on, the aim is to determine design parameters in a way that does not violate clearance but that considers how the joints are realised. Finally, kinematic design and design clarity both compare actual mobilities with intended mobilities and attempt to reduce incorrect constraints, therefore, the elemental interfaces approach can also be used to assess mobility at a system level.

### 3.2 Example case study

Applying the elemental interfaces approach to the glue gun, the digitalised closing type matrix for every joint has to be derived. Figure 7 shows the two types of joints and their matrix representations. The rotational joints allow for a rotation around z ($Rz, \tilde{R}z$ are 0); the prismatic joint allows for a translation in x direction ($x, \tilde{x}$ are 0). The bodies relate to each other in a serial chain, forming one kinematic loop.
Applying the boolean operations according to Roth, a particular movability in a serial relationship is locked at a system level when every joint of the chain (component level) locks movability as well.

<table>
<thead>
<tr>
<th>Component</th>
<th>Translation</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>WSP</td>
<td>Moved</td>
<td>Fixed</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
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<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>S</td>
<td>1</td>
</tr>
</tbody>
</table>

The system has two degrees of freedom (fitting with the results of the Kutzbach Grübler criterion). Additionally, the matrix contains information that translations and rotations are locked, differentiating each movability into positive and in the negative direction. In the glue gun example, translations in x direction and rotations about z are free.

\[ S = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}; F = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & M & M \end{pmatrix} \rightarrow S + F = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & M & M \end{pmatrix} \] (3)

If the rotation is now constrained through an additional moment (pulling the trigger of the glue gun), the translation of the carriage in x direction is the only remaining motion (Formula 3). This shows that results can be obtained that agree with the results of the Kutzbach Grübler Criterion.

4. Conclusions

The Kutzbach-Grübler criterion and the elemental interfaces approach generally provide a way to analyse system movability. The differences lie in the depth of detail in the results, the knowledge required to obtain a useful solution, and the effort required to obtain the different procedures. Table 2...
contains an assessment of both procedures. The main insight is that the approaches complement one another.

Table 2. Comparison of the Kutzbach Grübler criterion and the elemental interfaces approach

<table>
<thead>
<tr>
<th>General applicability</th>
<th>Kutzbach Grübler Criterion</th>
<th>Elemental interfaces approach</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>+</td>
<td>- Unknown dependent degrees of freedom lead to incorrect results of the Kutzbach Grübler criterion. - Unknown type of mechanism (3d, plane, spherical) lead to incorrect results of the Kutzbach Grübler criterion. - Eccentrical interfaces lead to incorrect results of the elemental interfaces approach.</td>
</tr>
</tbody>
</table>

| Consistency with Design for Clarity | -                           | +                             | - Results of the Kutzbach Grübler criterion cannot be directly used for a design for clarity analysis. - The elemental interfaces approach can be applied in the same way to analyse system movability or clarity. |

| Time effort | +                           | -                             | - The Kutzbach Grübler criterion is very quick to apply if the type of mechanism and dependant degrees of freedom are known. - The elemental interfaces approach in its actual form is time consuming. |

| Depth of Detail | -                           | +                             | - The Kutzbach Grübler criterion only contains information on mobility. For further measures, an additional analysis must show which translations and rotations are free and which are locked. - The elemental interfaces approach directly indicates free and locked translations and rotations; measures can be identified easily. |

Where a system is generally known (type of mechanism) and there is sufficient information on movability for the current task or decision making, applying the Kutzbach Grübler criterion is the best procedure for analysing the system. If the results are not based on certain knowledge, the elemental interface approach is the better way to understand the system. Its major shortfall is that it is time consuming. Additionally, the results must be interpreted to avoid failures due to eccentrical relations between parts. The biggest advantage is that the same procedure can be used to assess system clarity and that it directly contains information about coordinates. In combination with a matrix-based force/moment representation, it is easy to derive solutions that correctly constrain a mechanism. Therefore, the elemental interfaces approach also supports the synthesis step.

Further development must aim for computer-based support of the elemental interface approach to increase its usability and decrease analysis time. Additionally, the general applicability of the approach must be justified in further case studies.

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References


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DESIGN METHODS