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STACK-UP ANALYSIS OF STATISTICAL TOLERANCE INDICES FOR LINEAR FUNCTION MODEL USING MONTE CARLO SIMULATION

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Abstract

Tolerancing is important in the mechanical design process because it affects product quality and manufacturing cost. Various tolerancing methods have been studied while considering quality and cost of a product. However, tolerance for design element is rounded to one scalar value, even though designers decide the value statistically considering machining error. Therefore, a next generation tolerancing method is required. Fortunately, a useful tool called statistical tolerance index is available. This tool limits design drawing process capability indices on design drawing, so that a manufacture process may satisfy this limitation. To decide the limitation suitably, a stack-up problem of statistical tolerance indices is formulated like a problem of conventional tolerance analysis. The stack-up problem can be represented by Minkowski-sum on a hyper-plane of the mean and the standard deviation square. Therefore, the problem can be numerically solved using the convex envelope algorithm and Monte Carlo simulation. We first begin the study by analysing the problem using Monte Carlo simulation.

Keywords: Tolerancing, Process Capability Index, Stack-up analysis, Tolerance representation and management, Computational design synthesis

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1 INTRODUCTION

Actual dimensions of machined parts do not match a nominal dimension specified at design stage because of machining error. A mechanical product consists of the machined parts, and the product performance depends on a functional dimension, which results from stack-up of the parts. Because performance and quality of products are variable, dimension of each part is managed by a tolerance at design stage. If the tolerance value is tight, performance variation will reduce but manufacturing cost increase. Therefore, tolerancing is an important task in a detailed design process, which is one of downstream process in mechanical design process. Tolerancing method is generally classified in worst-case and statistical methods (1988, Chase). The worst-case method is traditional and easy to calculate, in which stack-up of the parts variations is modeled as a sum of limit of each part variation. Although the method perfectly guarantees an interchangeability of the parts, the specified tolerances tend to be tight. On the other hand, the statistical method can relax the tolerances considering statistical distributions of the parts dimensions. The method is based on statistic rule, so that it is compatible with mass production. There are various studies of the statistical methods.

For example, Skowronski and Turner (1997) examined Monte-Carlo variance reduction techniques, importance sampling and correlation. They also proposed a method for using them in statistical tolerance synthesis. Gao and Huang (2003) proposed an optimal tolerance balancing method for a nonlinear model. The method was based on process capability and validated through tests. Maurice et al. (2005) proposed weighted inertia tolerance to obtain the best possible compromise between statistical and worst-case tolerancing methods. The tolerance principle involved calculating the allowable range of the mean square deviation in relationship to the target. Zhang et al. (1998) proposed PCI (process capability index) -based tolerance as a predetermined statistical tolerance zone. This tolerancing method can be used as an interface between design specification and statistical process control. In quality engineering, Li (2000) studied the relationship between unbalanced tolerance design and quality loss function. The study concluded that the optimal setting of the process mean that minimizes the expected quality loss was obtained with respect to the asymmetrical ratio. Choi et al. (1999) applied a complex search method to ensure an optimal allocation when tolerance limits were used and when Taguchi's quadratic loss function was defined. The aim of those studies was to calculate reasonable tolerances.

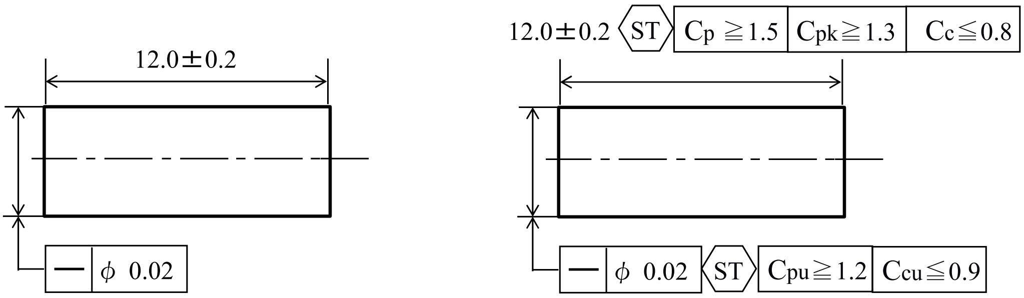
Although designers use the statistical method considering a condition of product performance and manufacturing cost, the conditions are rounded to the conventional tolerances as a scalar value. If more details information are added to the tolerances on design drawing, intention of the designer can be reflected into actual products. Consequently, an additional value is given to the design drawing and the products. Fortunately, a useful tolerance specification, which we called statistical tolerance index (STI), has been standardized in ASME Y14.5. The STI, in which process capability index is limited, is a specification for mass production. When the STI is specified in design drawings, manufacturing processes must satisfy the limitation of the STI under statistical process control. Although the STI might cause an additional manufacturing cost, that is not demerit if an advantage of the STI is superior to the disadvantage.

There are two main problems before applying the STI to actual design process because a product generally consists of several parts. Those are similar to tolerance stack-up and tolerance allocation in conventional tolerancing. Before allocating the STI into parts dimensions reasonably, STI stack-up problem should be researched and clarified. From the result of previous studies, the problem was known to be complex and difficult even if a product consists of two parts. Srinivasan et al. (1997) proved that a solution of the problem was generally represented by the Minkowski-sum on the hyper-plane of mean and square of standard deviation. They showed an algebraic solution for the problem under condition that only *Cpk* and *Cc* were specified. Based on their study, Otsuka (2012) derived more general algebraic solution for the problem and clarified the applicability condition of the solution. However, the solution is still not practical for actual product design because it requires many assumptions. Although an algebraic solution for the problem is useful in STI allocation problem, we predict that it is difficult to derive more general solution without removing the assumptions.

Therefore, a numerical method should be developed to obtain an alternative solution. Because the operators of the Minkowski-sum and the convex envelope are mathematically commutative, the STI stack-up problem can be approximately solved using the convex envelope algorithm and Monte Carlo simulation. First of all, a numerical method using Monte Carlo simulation for the STI stack-up problem is proposed in this paper. In case study, the proposed method is evaluated for a virtual assembly model consisting of four parts with linear stack-up.

2 STATISTICAL TOLERANCE INDEX

STI is one of specification using a process capability index (PCI) as an additional indicator for a manufacturing process with a conventional tolerance. STIs can be specified adding uppercase 'ST' into hexagon. Figure 1(a) shows a design drawing that conventional tolerances are specified. Figure 1(b) shows a design drawing that the conventional tolerances and STIs are specified. The specified values of dimensions, tolerances and STIs in Figure 1 are examples. STI is applicable to both dimension and geometrical tolerances, so that STI can limit the distribution parameters of dimensional or geometrical errors of machined parts lot-by-lot. The STI is a useful tool to control performance and quality of mass produced products. However, STI is still not common in a current design process because of its complexity.



(a) with conventional tolerance *(b) with statistical tolerance indices*

Figure 1. Examples of mechanical design drawing

2.1 Process capability index

In mass production, machining processes must be controlled lot-by-lot due to machining errors. To evaluate the machining process quantitatively, PCIs have been used for a long time, which are non-dimensional parameters defined as follows (1999, Srinivasan).

$$Cp = \frac{U-L}{6\sigma} \tag{1}$$

$$Cpk = \min(C_{pl}, C_{pu}) \quad \text{where} \quad Cpl = \frac{\mu - L}{3\sigma}, \; Cpu = \frac{U-\mu}{3\sigma} \tag{2}$$

$$Cc = \max(C_{cl}, C_{cu}) \quad \text{where} \quad Ccl = \frac{\tau-\mu}{\tau-L}, \; Ccu = \frac{\mu-\tau}{U-\tau} \tag{3}$$

$$Cpm = \frac{U-L}{6\sqrt{\sigma^2 + (\mu-\tau)^2}} \tag{4}$$

Where L, U, μ, σ and τ are lower limit of size, upper limit of size, tolerance, process mean, process standard deviation and target dimension, respectively. T is tolerance defined by a difference of U and L. In order to apply PCIs to an unilateral tolerance, the additional indices *Cpl* and *Cpu* or *Ccl* and *Ccu* can be used instead of *Cpk* or *Cc*. When PCIs are limited within certain specified values such as $Cp \geq p$, $Cpk \geq k$, $Cpl \geq kl$, $Cpu \geq ku$, $Cc \leq c$, $Ccl \leq cl$, $Ccu \leq cu$ and $Cpm \geq m$ where p, k, kl, ku, c, cl, cu and m are design parameters, the process must be controlled to keep its capability within each parameter range. As the PCIs are defined by process mean and/or standard deviation, the process assumed to be controlled under statistical process control. When several STIs are specified simultaneously on a dimension, the allowable range of the mean and the standard deviation are hard to understand. To understand the range easily, a population parameter zone is introduced.

2.2 Population parameter zone and statistical tolerance index

The population parameter zone (PPZ) represents an allowable range on $\mu - \sigma$ plane when STIs are specified. Figure 2 (a)-(d) shows examples of PPZ when a certain STI is specified. Figure 2 (e)-(h) shows examples of PPZ when multiple STIs are specified.

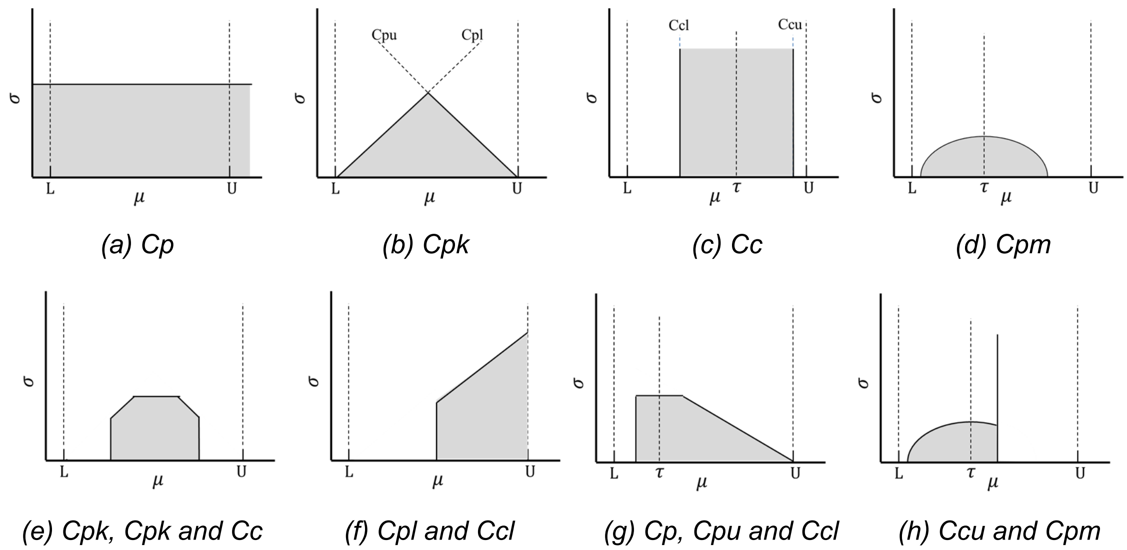


Figure 2. Overview of restriction of STI on population parameter zone

The horizontal axis is mean value, and the vertical axis is standard deviation value. The painted area is allowable range of the mean and the standard deviation, and the solid boundary of the range is an allowable limit. When STIs are specified for each part dimension, the dimension has each PPZ respectively. In the design process, designers need to decide suitable values and kinds of STI for each part dimension based on required function, performance and cost of a final product. The decision process is equivalent to a tolerance allocation. STI stack-up problem must be solved before discussing STI allocation problem, because the allocation problem is an inverse problem of the stack-up problem.

3 STACK-UP ANALYSIS OF STATISTICAL TOLERANCE INDEX

3.1 Previous work

Functionality and performance of a final product usually depend on a functional dimension of assembly consisting of several parts. In the design process of the final product, constraints of the functional dimension are first decided based on customer and supplier needs. Subsequently, designers allocate the constraints to parts tolerances and specify it on design drawing as shown in Figure 1. The allocation process is carried out based on a solution of a tolerance stack-up problem.

There are many studies of the stack-up problem for the conventional tolerance using worst-case and statistical methods. On the other hand, there are few researches about STI stack-up problem. Srinivasan et al. (1997) proved that a solution of the problem was represented by the Minkowski-sum on $\mu - \sigma^2$ plane. They also showed an algebraic solution for the STI stack-up problem when *Cpk* and *Cc* are specified and when the values of *k* and *c* for each part are same. However, it is not practical use because of a lack of degree of design freedom. Furthermore, calculation details of the Minkowski-sum were not shown in the report. In this paper, a numerical method using Monte Carlo simulation is proposed for the STI stack-up. A merit of the proposed method is applicability for more general situations of STI stack-up.

3.2 Model of STI stack-up problem

A functional dimension depends on parts dimensions x_i. In general, a functional dimension X consisting of n parts can be written as following,

$$X = h(x_1, x_2, ..., x_n) \quad (5)$$

where i is the parts identifier, and h can be linear or nonlinear. Figure 3 shows an assembly model used in this study. The assembly consists of four parts, in which h is assumed be linear. The functional dimension of the assembly is given as follows.

$$X = \sum_{i=1}^{4} x_i \quad (6)$$

If the parts dimensions are independent each other such as the model, the mean and the standard deviation of the functional dimension of mass produced assemblies, μ_X and σ_X, can be calculated based on statistical rules as follows.

$$\mu_X = \sum_{i=1}^{4} \mu_i \quad (7)$$

$$\sigma_X = \sqrt{\sum_{i=1}^{4} \sigma_i^2} \quad (8)$$

Note that, an assumption of a normal distribution is not required because the equations (7) and (8) hold regardless of distributions of the parts dimensions.

When STIs are specified on each part dimension with conventional tolerance, the PPZs can be generated on the $\mu_i - \sigma_i$ planes, respectively. Accordingly, PPZ of the functional dimension is spontaneously projected on the $\mu_X - \sigma_X$ plane. The PPZ on the $\mu_X - \sigma_X$ plane is a solution of the STI stack-up problem. A boundary of the PPZ is an important element because it is a limit of the mean and standard deviation. Calculation of the boundary is achieved by discretizing μ_X and gathering solutions of maximum optimization problems for each μ_X, in which an objective function and constraints are equation (8) and STIs, respectively. An algebraic equation of the solution has established under some conditions.

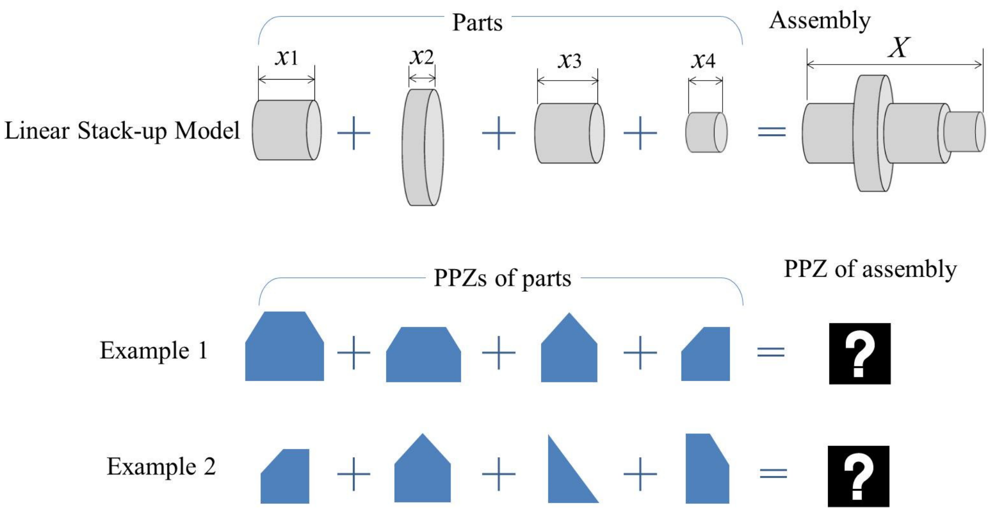


Figure 3. Illustrations of assembly model and Stack-up problem

3.3 Algebraic solution under conditions

In our previous study (Otsuka, 2012), a solution of STI stack-up problem is algebraically expressed under some conditions, in which the STI specifications are specified using only *Cpk* and *Cc*, and the following conditions must be held.

$$T_1/k_1 \geq T_2/k_2 \geq T_3/k_3 \geq T_4/k_4 \tag{9}$$

$$T_1(1-c_1)/k_1^2 \geq T_2(1-c_2)/k_2^2 \geq T_3(1-c_3)/k_3^2 \geq T_4(1-c_4)/k_4^2 \tag{10}$$

If the functional dimension is linear, such as equation (6), parts identifiers are exchangeable without loss of generality. In other word, the identifiers in equation (6) do not need to match the ones of equations (9) and (10). If both conditions are satisfied, the solution of STI stack-up problem is given by a set of curves as follows.

$$\sigma_X^j \leq \sqrt{\sum_{i=1}^{j-1}\left(\frac{T_i}{6k_i}\right)^2 + \sum_{i=j+1}^{n}\left(\frac{T_i(1-c_i)}{6k_i}\right)^2 + \left(\frac{1}{3k_j}\right)^2\left[\mu_X^j - \left(L'_X - \frac{T_j}{2} + \sum_{i=1}^{j}\frac{T_i c_i}{2}\right)\right]^2} \tag{11}$$

Domains for each j is given by

$$L'_X + \sum_{i=1}^{j-1}\frac{T_i c_i}{2} \leq \mu_X^j \leq L'_X + \sum_{i=1}^{j}\frac{T_i c_i}{2} \tag{12}$$

where n is the number of parts (in this study, n=4). L'_X is the mean value given by as follow.

$$L'_X = \left(U_X + L_X - \sum_{i=1}^{n} T_i c_i\right)\Big/2 \tag{13}$$

The equation (11) is defined only within the left side of PPZ because the solution is confirmed to be linear symmetry for $\mu_X = (U_X + L_X)/2$. Additionally, the stack-up solution has also been solved algebraically when only *Cpm* is specified on parts (Otsuka, 2011). In this paper, the details of the results are omitted for lack of space.

If a solution of the STI stack-up problem is given as an algebraic equation, it can be easily used in an STI allocation problem as constraints. Otsuka et al. (2014) have proposed the STI allocation method using genetic algorithms to design appropriate STIs parameters for a virtual product model consisting of five parts under the assumption that only *Cpk* and *Cc* are specified. However, if several kinds of STI are specified for each part simultaneously or if unilateral tolerance such as geometrical tolerance is specified, it is difficult to derive such the algebraic equation, and the solution will be more complex form. Therefore, an alternative method is needed to obtain the solution.

3.4 Proposed method using monte carlo simulation

To solve general STI stack-up problems, a method using Monte Carlo simulation is proposed. From the view point of part, a boundary on PPZ of a part is the worst-case mean and standard deviation. Therefore, the boundaries of parts should be focused to obtain the worst-case solution of assembly. Note that, the word "worst-case" means a worst situation about mean and standard deviation. Figure 4 shows the algorithm of the proposed method. In the first procedure, uniform random number about mean of each part is generated within range limited by *Cc*, *Ccl* and/or *Ccu*. Minimum standard deviation of each PPZ is calculated based on the random number and other specified STIs, such as *Cp*, *Cpk* and *Cpm*. Then, mean and standard deviation of assembly are calculated by Eqs. (7) and (8) using the random number and the minimum standard deviation, respectively. A point cloud is obtained by repeating those processes. The PPZ boundary of assembly can be estimated based on the point cloud.

The proposed method can be applicable if the equation (5) is clearly known such as equation (6) and if the parts dimensions are independent each other. Because Monte Carlo simulation is probabilistic method, an approximation error exists. Although the accuracy of the solution might depend on trial number of Monte Carlo simulation, we do not discuss the number of the simulation in this study. Note that, an assumption of a normal distribution for the part dimension is not required in this method.

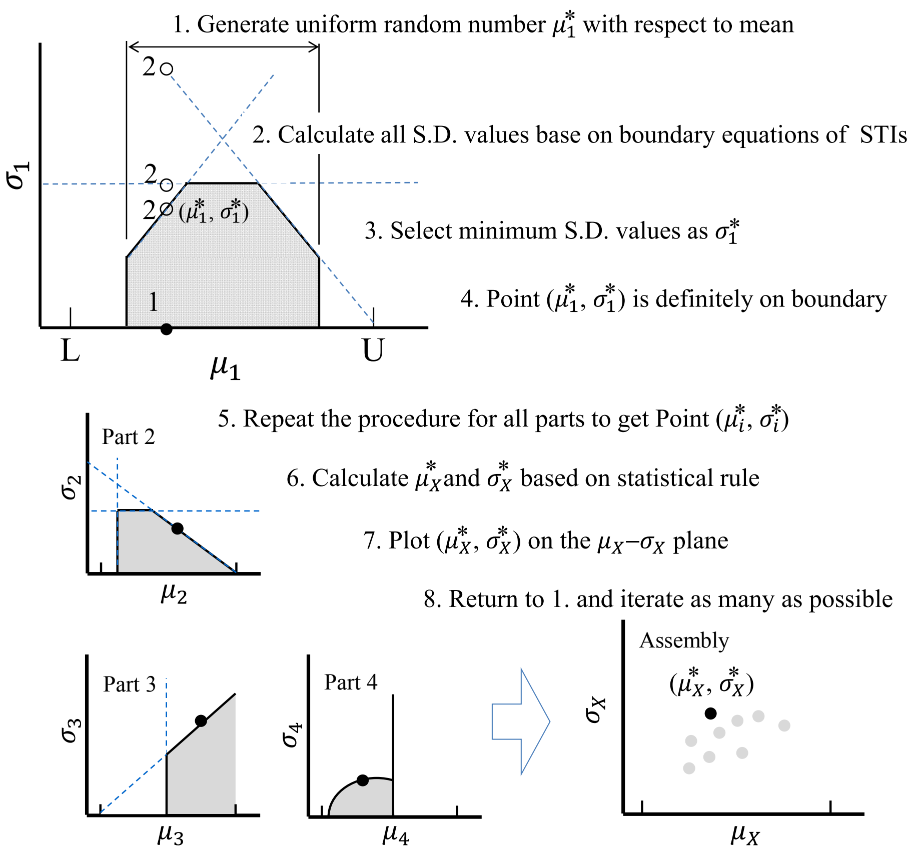


Figure 4. Flowchart of analysis method using Monte Carlo simulation

4 CASE STUDY

4.1 Confirmation of the proposed method

First, we tested the proposed algorithm with a certain situation of which the theoretical solution is given by equation (11). A trial number of Monte carlo simulation was set at 1,000,000 to obtain a detailed result. Dimensions and tolerances of a virtual product model are given in Table 1, in which units of the parameters are the millimetres. The tentative STI parameters, which satisfy Eqs. (9) and (10), are given in Table 2. The functional dimension X of the assembly is assumed to be the linear stack-up model given by equation (6). Figure 5 shows the boundaries of the PPZs of parts dimensions and point cloud calculated by the proposed method.

Table 1. Basic parameters of assembly model

Parts identifier	1	2	3	4		Assembly
Dimension : x	12.00	6.00	10.00	8.00		$X = 36.00$
Tolerance : T	0.10	0.04	0.08	0.06		0.28

Table 2. STI parameters for algorithm test

Parts identifier	1	2	3	4
Designed value of k	1.30	1.75	1.40	1.60
Designed value of c	0.88	0.80	0.87	0.85

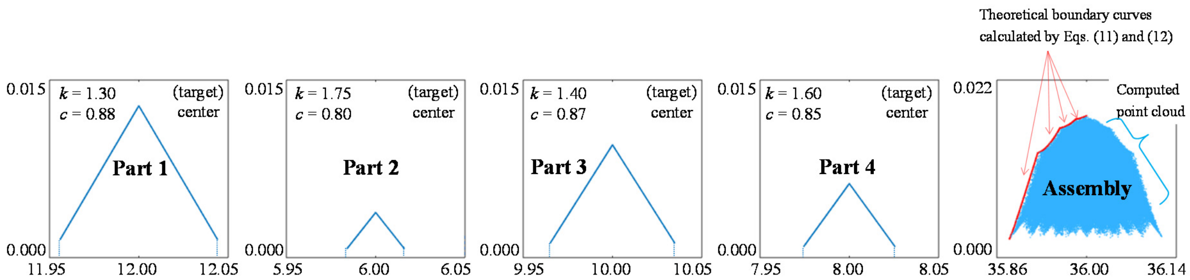


Figure 5. Comparison of theoretical solution and computed solution (Solid curves are theoretical boundary written by equation (9))

The curves in the graph on the right-side end are the theoretical solutions given by equation (11). The figure shows that all the computed points are plotted within the theoretical boundaries. This confirms that the proposed method correctly provided a numerical solution for the STI stack-up problem. In the next subsection, the method is applied to the same linear stack-up model when more STIs are specified complicatedly.

4.2 Application example for various cases

The proposed method is conducted for other cases, in which STI stack-up cannot be solved theoretically. In this paper, representative four cases are reported as shown in Figure 6.

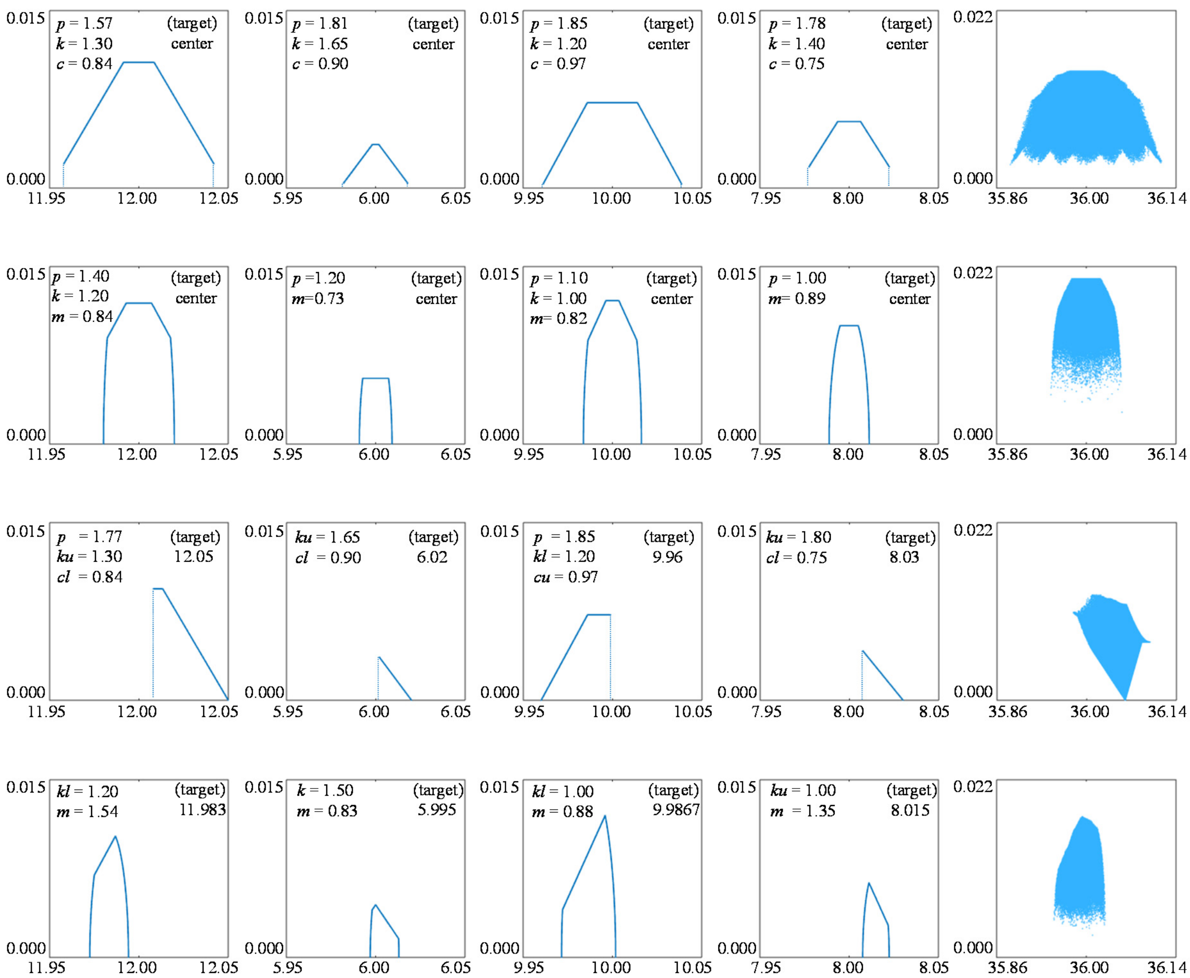


Figure 6. Case studies

Note that, the numerical solution in case study cannot be certificated because the theoretical solution does not exist at present. From the results of the case study, it confirmed that the proposed method provided a numerical solution for a general STI stack-up problem even when arbitrary kinds of PCI and its parameters are specified. Figure 6 also shows that the numerical solutions tend to be more complex when the target dimension is not tolerance center value. Because Monte Carlo simulation is a probabilistic method, sensitivity of the parameters, p, k, kl, ku, c, cl, cu, m and τ, with respect to the numerical solution should be evaluated.

5 CONCLUSION

In this study, a numerical method was proposed for STI stack-up problem. The method is based on the Monte Carlo simulation and statistics rule. In case study, the proposed method was applied to a virtual assembly model consisting of four parts with linear stack-up. First, the method was validated by applying it to a simple STI stack-up problem of which theoretical solution was known. Subsequently, the method was applied to some general cases, in which various STIs were specified on each part. Finally, the method provided a numerical solutions for the general STI stack-up problems.

For further study, accuracy analysis of the numerical solution obtained by the proposed method is planned because Monte Carlo simulation provides only a probabilistic solution. The proposed method will be also checked whether it is applicable when the function h is nonlinear. In the nonlinear model, the mean and the standard deviation of the functional dimension must be newly formalized based on statistics rules. Furthermore, the convex envelope method will be applied to the point cloud calculated by the proposed method, so that an approximate solution for general STI stack-up problem is obtained as set of line segments. After those studies, STI allocation method will be researched so that designers can specify appropriate kinds of STIs and its values on parts.

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