DESIGN FOR LIFECYCLE PROFIT WITH A SIMULTANEOUS CONSIDERATION OF INITIAL MANUFACTURING AND END-OF-LIFE REMANUFACTURING

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ABSTRACT
Remanufacturing is emerging as a promising solution for achieving green, profitable businesses. This paper considers a manufacturer that produces new products and also remanufactures products that become available at the end of their lifecycle. For such a manufacturer, design decisions determine both the initial profit from manufacturing and future profit from remanufacturing. To maximize the total profit, design decisions must carefully consider both manufacturing and end-of-life stages together. To help in the lifecycle design, this paper proposes a mathematical model using mixed integer programming. With an aim to maximize the total lifecycle profit (i.e., the sum of the profits from initial manufacturing and end-of-life remanufacturing), the proposed model identifies an optimal product design (i.e., design specifications and the selling price) for the new and remanufactured products. It optimizes both the initial design and design upgrades at the end-of-life stage and also provides corresponding production strategies. To illustrate, the developed model is demonstrated with an example of a desktop computer.

Keywords: lifecycle, remanufacturing, end-of-life, design for environment

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1 INTRODUCTION
As environmental regulations become increasingly stringent and people are more concerned about environmental issues, manufacturers are faced with the challenge of operating both green and profitable business. Remanufacturing is emerging as a promising solution to meet this challenge. In remanufacturing, products with a like-new condition are produced using parts retrieved from used and discarded products (hereinafter called end-of-life products). By utilizing the resources and value remaining in their end-of-life products, companies can reduce the amount of waste that must be disposed. Recently, manufacturers across a wide range of industries have turned to remanufacturing. Caterpillar, John Deere, Apple, Xerox, HP, and Sony are among the notable examples. As functional sales (such as leasing) and asset recovery services by manufacturers increase, remanufacturing is expected to become more popular and prevalent (Sundin and Bras, 2005; Zhao et al., 2010). Design is one of the most important considerations for successful remanufacturing. However, for a company which manufactures and sells both new and remanufactured products, optimizing product design is not a simple task. Design decisions made at the initial design stage affect both the profits from initial manufacturing and end-of-life remanufacturing. To maximize the total profit from the entire life cycle of a product, design decisions must be made by considering both stages together. Rapid changes in technology and customer preferences complicate the design decision even more. In a market with such rapid changes, initial product design determined at the manufacturing stage quickly becomes obsolete and outdated. To attract customers in the market, remanufactured products may need appropriate part upgrades. Therefore, product design must be optimized in a way that considers possible part upgrades at the end-of-life stage (Sand and Gu, 2006; Östlin et al., 2009; Kwak and Kim, 2013).
This paper considers a company that makes and sells new products and also sells remanufactured versions of the new products that become available at the end of their lifecycle. To help in optimal product design for the company, this paper proposes a mathematical model using mixed integer programming. The proposed model identifies the optimal product design and corresponding production strategies that maximize the total lifecycle profit. Here, lifecycle profit denotes the **sum of the profits from initial manufacturing and end-of-life remanufacturing.** To be more specific, the model optimizes the following decisions (Figure 1):
- Initial design (both specifications and selling price) and production quantity of the new product
- Number of units of used products to take back (or buy back) at the end-of-life stage
- Design upgrades and production quantity of the remanufactured product

The rest of the paper is organized as follows. Section 2 discusses the relevant literature, followed by the proposed mathematical model in Section 3. Section 4 illustrates the model with the example of a desktop computer. Section 5 summarizes the paper with future research directions.

![Figure 1. Two components of optimal product design for lifecycle profit: initial product design and design upgrade at the end-of-life stage](image)

2 LITERATURE REVIEW
Given the rapid changes in technology and customer preferences, optimal design is a critical success factor for manufacturers. To compete in the market, manufacturers need to identify optimal specifications and selling prices for their products. In the engineering design community, such optimal
design has been discussed focusing on new product sales. Design for market systems (DMS) and decision-based design (DBD) are well-known streams of research to this end. Various approaches have been proposed to optimize new product sales, including by Hazelrigg (1998), Wassenaar et al. (2003), Gu et al. (2002), Kumar et al. (2006), and Frischknecht et al. (2010). When remanufacturing is involved, design optimization encompasses additional decisions on part reuse and upgrades at the end-of-life stage. The decisions include: (1) whether to reuse a part or upgrade; and (2) the new specification of a part when it is to be upgraded. Despite growing interest in remanufacturing, only a few studies have made progress concerning an optimal design of this sort that also considers upgrades. Tsubouchi and Takata (2007) presented a model for determining the optimal timing and content of module-based design upgrades. The model attempted to satisfy customers’ requirements, while minimizing the environmental load from production. Rachaniotis and Pappis (2008) proposed a decision making model for remanufacturing a set of systems, in which the parts deteriorated at different rates and had different levels of importance for the system. The model determined which parts should be reused, replaced, upgraded, or disposed in order to maximize the performance of the overall systems. Chung et al. (2010) presented a dynamic programming model for

### Table 1. Mathematical notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi_N, \Pi_R$</td>
<td>Profit from selling the new and the remanufactured products, respectively</td>
</tr>
<tr>
<td>$x_{Nt}, x_{Rt}$</td>
<td>Specification of part $i$ of the new and the remanufactured products, respectively</td>
</tr>
<tr>
<td>$p_N, p_R$</td>
<td>Selling price of the new and the remanufactured products, respectively</td>
</tr>
<tr>
<td>$\beta_N, \beta_R$</td>
<td>Production amount for the new and the remanufactured products, respectively</td>
</tr>
<tr>
<td>$D_N, D_R$</td>
<td>Demand for the new and the remanufactured products, respectively</td>
</tr>
<tr>
<td>$t$</td>
<td>Product end-of-life year; time when the product returns for remanufacturing</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Take-back rate at year $t$</td>
</tr>
<tr>
<td>$S_R$</td>
<td>Supply of the end-of-life product at year $t$</td>
</tr>
<tr>
<td>$y_i$</td>
<td>Binary variable indicating whether part $i$ of the remanufactured product maintains its original specification ($x_i=1$) or upgrades its specification ($x_i=0$)</td>
</tr>
<tr>
<td>$\delta_i(t)$</td>
<td>Generational difference of part $i$ of the end-of-life product at year $t$</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Generational difference of part $i$ being newly decided when the part $i$ is to be upgraded</td>
</tr>
<tr>
<td>$l_i$</td>
<td>Binary variable indicating whether part $i$ needs new part purchase (=1) or not (=0)</td>
</tr>
<tr>
<td>$R_i(t)$</td>
<td>Number of units of reusable part $i$ at year $t$</td>
</tr>
<tr>
<td>$R_i(t)$</td>
<td>Reusability of part $i$ of the end-of-life product at year $t$</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>Average frequency per year in which a successive generation of part $i$ newly released</td>
</tr>
<tr>
<td>$C_{Nt}^{\text{part}}, C_{Rt}^{\text{part}}$</td>
<td>Cost of purchasing (or manufacturing) parts for the new and the remanufactured products, respectively</td>
</tr>
<tr>
<td>$C_{Nt}^{\text{market}}, C_{Rt}^{\text{market}}$</td>
<td>Cost of assembling and distributing the new and the remanufactured products, respectively</td>
</tr>
<tr>
<td>$C_{\text{takeback}}, C_{\text{recond}}$</td>
<td>Cost of take-back and reconditioning, respectively</td>
</tr>
<tr>
<td>$V_i^{\text{new}}(x_i)$</td>
<td>Market value of purchasing a new part $i$ when the part’s specification is $x_i$</td>
</tr>
<tr>
<td>$V_i^{\text{recycled}}(x_i)$</td>
<td>Market value of recycling a used part $i$ when the part’s specification is $x_i$</td>
</tr>
<tr>
<td>$M_{\text{recycle}}$</td>
<td>Revenue from recycling (i.e., material recovery)</td>
</tr>
<tr>
<td>$c_{\text{takeback}}$</td>
<td>Unit cost of taking back (buying back) the end-of-life product at year $t$</td>
</tr>
<tr>
<td>$c_{\text{recond}}$</td>
<td>Unit cost of reconditioning operations for a reusable part $i$</td>
</tr>
<tr>
<td>$c_{\text{market}}$</td>
<td>Unit cost of assembling and distributing a product</td>
</tr>
<tr>
<td>$Q_N, Q_R$</td>
<td>Market size for the new and the remanufactured products, respectively</td>
</tr>
<tr>
<td>$M$</td>
<td>Big M; a very large positive number</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Annual interest rate</td>
</tr>
<tr>
<td>$G_{\text{max}}^N, G_{\text{max}}^R$</td>
<td>Maximum value that the generational difference of part $i$ can have for the new and the remanufactured products, respectively</td>
</tr>
</tbody>
</table>
determining the optimal upgrade plan for an existing product. Assuming product users as the decision maker, the proposed model identified the timing and content of upgrades that meet future performance requirements with a minimum cost. Kwak and Kim (2013) proposed a model for market positioning of a remanufactured product. When the design of a new product is given, the model optimized the design and selling price of the remanufactured product, considering possible upgrades of constituent parts. One limitation of the previous methods is that design influences on the initial manufacturing and end-of-life remanufacturing have been considered separately. Product design determines not only the initial profit from the manufacturing stage, but also affects the future profit at the end-of-life stage (i.e., remanufacturing). Previous approaches, however, have only focused on improving one of the stages, but not the stages together. Exceptions can be found in Zhao and Thurston (2010) and Ma et al. (2012). They developed a mathematical model to determine an optimal product design that maximizes the profits from both initial sales and end-of-life recovery. They showed that the total profit can be maximized when both ends of the product lifecycle are considered at the same time. However, they did not incorporate part upgrades at the end-of-life stage.

The current design model presented in the next section provides a simultaneous consideration of profits both from initial manufacturing and end-of-life remanufacturing with optimal part upgrade decisions. The model can identify optimal designs for two different sets of products – new and remanufactured, while the identity of product is maintained. For example, two sets of same type consumer electronics products are designed, wherein the details of product specs are different for new and remanufactured. The details of the model follow in the next section.

3 OPTIMAL PRODUCT DESIGN FOR LIFECYCLE PROFIT

This section proposes a mathematical model for optimal product design. Table 1 presents the mathematical notations used in the model. Using mixed integer programming, the proposed model identifies optimal specifications, selling prices, and the corresponding production strategies for both new and remanufactured products. The goal of the model is to maximize the total lifecycle profit, i.e., the sum of the profits from initial manufacturing and end-of-life remanufacturing.

The proposed model is based on the following assumptions. First, the decision maker has no other products in the target market, so there is no risk of cannibalization. Second, the product to be remanufactured has a modular structure, and upgrades are made through part replacement. Third, remanufacturing is instantaneous. Remanufacturing operations have a negligible lead time. Fourth, all non-reusable and leftover parts are transferred to third-party recyclers for material recovery. Lastly, the decision maker has good knowledge of the required inputs at the time of applying the model. How to estimate input values is left out of the scope of this study.

3.1 Part obsolescence and upgrade decisions

To represent product specifications and its technological obsolescence, this study uses the concept of generational difference (Kwak and Kim 2013). As product technology advances, cutting-edge parts of a new generation start to appear in the market. In this study, the newer part corresponds to the greater number of generations, and the cutting-edge part corresponds to the maximum generation (the latest). Then, the generational difference of a part is the gap between its generation and the current maximum generation of the cutting-edge part. (For example, a product consisting of cutting-edge parts only has zero generational differences for each and every part.) Therefore, the generational difference indicates, in terms of the technology, how old an existing part is compared to the cutting-edge part.

![Figure 2. Possible decisions on part upgrades and their implications in remanufacturing](https://via.placeholder.com/150)

<table>
<thead>
<tr>
<th>Decision</th>
<th>Condition</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upgrade</td>
<td>( \beta_x &lt; R(t) ) (( \beta_x \leq R(t) ))</td>
<td>Purchase ( \beta_x ) units with ( x_{\beta} = u_i )</td>
</tr>
<tr>
<td>Upgrade</td>
<td>( \beta_x &gt; R(t) ) (( \beta_x = R(t) ))</td>
<td>Reuse ( \beta_x ) units</td>
</tr>
<tr>
<td>Upgrade</td>
<td>( \beta_x \leq R(t) ) (( \beta_x &gt; R(t) ))</td>
<td>Reuse ( \beta_x ) units</td>
</tr>
</tbody>
</table>

Assuming product users as the decision maker, the proposed model identified the timing and content of upgrades that meet future performance requirements with a minimum cost. Kwak and Kim (2013) proposed a model for market positioning of a remanufactured product. When the design of a new product is given, the model optimized the design and selling price of the remanufactured product, considering possible upgrades of constituent parts. One limitation of the previous methods is that design influences on the initial manufacturing and end-of-life remanufacturing have been considered separately. Product design determines not only the initial profit from the manufacturing stage, but also affects the future profit at the end-of-life stage (i.e., remanufacturing). Previous approaches, however, have only focused on improving one of the stages, but not the stages together. Exceptions can be found in Zhao and Thurston (2010) and Ma et al. (2012). They developed a mathematical model to determine an optimal product design that maximizes the profits from both initial sales and end-of-life recovery. They showed that the total profit can be maximized when both ends of the product lifecycle are considered at the same time. However, they did not incorporate part upgrades at the end-of-life stage.

The current design model presented in the next section provides a simultaneous consideration of profits both from initial manufacturing and end-of-life remanufacturing with optimal part upgrade decisions. The model can identify optimal designs for two different sets of products – new and remanufactured, while the identity of product is maintained. For example, two sets of same type consumer electronics products are designed, wherein the details of product specs are different for new and remanufactured. The details of the model follow in the next section.
In the current model, \(x_0\) denotes the specification of part \(i\) of a new product. It is represented in terms of the part’s generational difference at the manufacturing stage. As the specification becomes obsolete over time, the generational difference of part \(i\) increases with an annual average rate of \(\mu_i\); \(t\) years later (when the product reaches the end-of-life stage), the generational difference becomes \(\delta(t)\) which is equal to \(\text{floor}(x_0 + \mu_i \cdot t)\), i.e., the greatest integer less than or equal to \(x_0 + \mu_i \cdot t\). Given \(\delta(t)\), the specification of the remanufactured product \(x_R\) is defined as a function of \(\delta(t)\), i.e., \(x_R = \delta(t) \cdot y_i + u_i\), where \(y_i\) and \(u_i\) represent the decisions on part reuse and upgrading, respectively.

Figure 2 describes how decisions on part reuse and upgrading affect the remanufacturing operation, more specifically, what and how many used parts are reused and what and how many spare parts are purchased. If part \(i\) is determined to be upgraded \((y_i = 0)\), no parts are reused in remanufacturing. All \(R_i(t)\) units of reusable part \(i\) are sold to third-party recyclers for material recovery, while \(\beta_R\) units of a spare part with an upgraded specification \(u_i\) are newly purchased. If part \(i\) is determined to maintain its original specification \((y_i = 1)\), the next question is whether the \(R_i(t)\) units of reusable part \(i\) are sufficient to meet the production amount \(\beta_R\). If part \(i\) is insufficient in quantity for remanufacturing \((\beta_R > R_i(t); l_i = 1)\), spare parts that are new but having the original specification, are purchased for as many as \((\beta_R - R_i(t))\). In contrast, if there are enough reusable parts \((\beta_R \leq R_i(t); l_i = 0)\), only \(\beta_R\) units are used in remanufacturing while the rest \((R_i(t) - \beta_R)\) units are sent to third-party recyclers.

3.2 Remanufacturing process

The primary goal of remanufacturing is to retrieve valuable parts from end-of-life products and use them to produce marketable products. Remanufacturing typically involves two sequential activities: product take-back and a reprocessing operation. Figure 3 depicts the remanufacturing process considered in this paper and how the process is linked with new product sales.

Product take-back is the process of collecting (buying back) end-of-life products. Since product take-back determines the quality and quantity of feedstock processed later in the reprocessing operation, a key aspect of this activity is to determine how many products should be acquired. The current model assumes that, an \(\alpha\) fraction of the total new product sales \(\beta_N\) is taken back for remanufacturing at the end-of-life stage. Here, the take-back rate \(\alpha\) is one of the decision variables to optimize.
After product take-back, the collected products pass through a reprocessing operation. In the first stage of reprocessing, products are disassembled into a set of parts, and the resultant parts start their recovery as independent units. Two recovery options are considered for each part, i.e., reuse for product remanufacturing or material recycling. An important point is that not all resulting parts are reusable, and only reusable parts are qualified for reuse. Also, as discussed in Section 3.1, upgrading decisions also affect which and how many parts are reused. For the parts to be reused, reconditioning (e.g., cleaning, lubricating) is conducted as needed. In the last stage, parts from the end-of-life products are reassembled into \( \beta_R \) units of remanufactured products. Again, upgrade decisions affect the type and amount of new parts to purchase, as shown in Figure 3. When there is a shortage of parts, new spare parts can be externally procured.

3.3 Mathematical model

The optimization model is formulated in Equations (1)-(6). The objective of this model (Equation (1)) is to maximize the total lifecycle profit, where the lifecycle profit is the sum of two components: the profit from selling new products, \( \Pi_N \), and the profit from selling remanufactured products, \( \Pi_R \). The profit from remanufacturing is discounted with an annual interest rate of \( \theta \).

The profit from new product sales consists of three parts: the revenue from selling \( \beta_N \) units of new products (i.e., \( p_N \cdot \beta_N \)), the cost of purchasing (or manufacturing) parts for making \( \beta_N \) products (i.e., \( C_{\text{part}}^N \)), and the cost of assembling and distributing \( \beta_N \) products (i.e., \( C_{\text{market}}^N \)). The profit from remanufacturing consists of six components: the revenue from selling \( \beta_R \) units of remanufactured products (i.e., \( p_R \cdot \beta_R \)), the revenue from selling non-reusable or left-over parts to third-party recyclers (i.e., \( M_{\text{recycle}}^R \)), the cost of taking back \( S_R \) units of end-of-life products (i.e., \( C_{\text{takeback}}^R \)), the cost of acquiring parts for making \( \beta_R \) products (i.e., \( C_{\text{part}}^R \)), the cost of reconditioning reusable parts (i.e., \( C_{\text{recycle}}^R \)), and the cost of assembling and distributing \( \beta_R \) products (i.e., \( C_{\text{market}}^R \)).

\[
\begin{align*}
\text{maximize} & \quad \Pi_N + (1 + \theta)^{-1} \cdot \Pi_R \\
\text{subject to} & \quad x_{Ni} \cdot p_N \cdot \beta_N \cdot x_{Ri} \cdot p_R \cdot \beta_R, \quad \alpha, \quad y_i, \quad l_i, \quad u_i, \quad \delta_i(t) \\
\text{where} & \quad \\
& \quad \Pi_N = p_N \cdot \beta_N - (C_{\text{part}}^N + C_{\text{market}}^N) \\
& \quad \Pi_R = p_R \cdot \beta_R + M_{\text{recycle}}^R - (C_{\text{takeback}}^R + C_{\text{part}}^R + C_{\text{recycle}}^R + C_{\text{market}}^R) \\
& \quad C_{\text{part}}^N = \beta_N \cdot \sum_{i \in I} V_i^{\text{new}}(x_{Ni}) \\
& \quad C_{\text{market}}^N = C_{\text{market}}^N \cdot \beta_N \\
& \quad C_{\text{takeback}}^R = C_{\text{takeback}}^R \cdot S_R \\
& \quad C_{\text{part}}^R = \sum_{i \in I} [1 - y_i \cdot l_i \cdot (\beta_R - R_i(t))] \cdot V_i^{\text{new}}(x_{Ri}) \\
& \quad C_{\text{recycle}}^R = \sum_{i \in I} \beta_R \cdot h_i \cdot R_i(t) + y_i \cdot (1 - l_i) \cdot \cdot \beta_R \cdot V_i^{\text{new}}(x_{Ri}) \\
& \quad C_{\text{market}}^R = \cdot \beta_R \\
& \quad M_{\text{recycle}}^R = \sum_{i \in I} [S_R - y_i \cdot l_i \cdot R_i(t) - y_i \cdot (1 - l_i) \cdot \beta_R] \cdot V_i^{\text{new}}(x_{Ri}) 
\end{align*}
\]

Equations (2) through (6) formulate the constraints of the model. Equation (2) calculates the demand for the new and remanufactured products, i.e., \( D_N \) and \( D_R \). Product specifications, \( x_{Ni} \) and \( x_{Ri} \), and selling prices, \( p_N \) and \( p_R \), determine the size of the demand. The demand function can be defined through well-known demand modeling techniques, such as Discrete Choice Analysis (Wassenaar and Chen, 2003; Ben-Akiva and Lerman, 1985) and conjoint analysis (Green et al., 2001). This model also assumes that each part and the selling price have critical levels (i.e., \( \beta_{\text{max}}^N, \beta_{\text{max}}^R, p_{\text{max}}^N, p_{\text{max}}^R \)) for their values. In general, customers prefer lower generational differences and price. The critical levels represent the maximum generational differences and price that customers are willing to consider for purchasing the product. For example, if any part of a product has a generational difference greater than its critical value, then customers will not choose the product at all. Equation (2) prevents the generational differences and selling price from exceeding their critical values.
\[ D_N = f_N(x_N, p_N); D_R = f_R(x_R, p_R) \]
\[ x_{Ni} \leq \delta_{Ni}^{\max}; x_{ri} \leq \delta_{ri}^{\max}; p_N \leq p_N^{\max}; p_R \leq p_R^{\max} \]  \hspace{1cm} (2)

Equation (3) constrains the production quantity (or initial sales) \( \beta_N \) so as not to exceed the demand size \( D_N \). Unlike new production, remanufacturing is possible only when there exists both a supply of end-of-life products and demand for remanufactured products (Guide et al., 2003; Umeda et al. 2006). Thus, Equation (3) also constrains the production quantity \( \beta_R \) so as not to exceed the supply \( S_R \) or demand \( D_R \). As described in Section 3.2, the supply \( S_R \) is determined by the initial sales \( \beta_N \) and the take-back rate \( \alpha \).

\[ \beta_N \leq D_N; \beta_R \leq D_R; \beta_R \leq S_R \]
\[ S_R = \alpha \cdot \beta_N \]  \hspace{1cm} (3)

Equation (4) formulates decisions for part upgrades at the end-of-life stage. The variable \( x_{ri} \) denotes the generational difference of part \( i \) which is to be included in the remanufactured product. It is determined by two decision variables \( y_i, u_i \). When \( y_i = 0 \), a part upgrade is conducted, and the current part with \( \delta_i(t) \) is replaced by an upgraded part with \( u_i \). When \( y_i = 1 \), part \( i \) is reused, and at the same time, \( u_i \) becomes 0. Accordingly, \( x_{ri} \) equals \( \delta_i(t) \) which is the generational difference of the original part \( i \) at the end-of-life stage, i.e., \( \text{floor}(x_{ri} + \mu_i, t) \). The floor function is linearized in Equation (4) using a positive number \( \varepsilon \) less than 1 (e.g., 0.1 in this study).

\[ x_{ri} = \delta_i(t) \cdot y_i + u_i \; \forall i \]
\[ \delta_i(t) \leq x_{ri} + \mu_i \cdot t \; \forall i \]
\[ \delta_i(t) \geq (x_{ri} + \mu_i \cdot t) - 1 + \varepsilon \; \forall i \]
\[ \sum_{i \in J} y_i \cdot u_i = 0 \]  \hspace{1cm} (4)

Equation (5) considers if the available quantity of reusable part \( i \) (i.e., \( R_i(t) \)) is sufficient to produce \( \beta_R \) units of the remanufactured product. If part \( i \) is insufficient in quantity (i.e., \( \beta_R > R_i(t) \)), the indicator variable \( l_i \) becomes 1; in Equation (1), this implies that new parts as many as \( (\beta_R - R_i(t)) \) are purchased. Finally, Equation (6) represents variable conditions.

\[ R_i(t) = r_i(t) \cdot S_R \; \forall i \]
\[ \beta_R - R_i(t) \leq M \cdot l_i \; \forall i \]
\[ \beta_R - R_i(t) \geq M \cdot (l_i - 1) \; \forall i \]  \hspace{1cm} (5)
\[ \beta_N, \beta_R, x_{ni}, x_{ri}, u_i, \delta_i(t) \in \text{nonnegative integer}; \; y_i, l_i \in \{0, 1\} \; \forall i \]
\[ 0 \leq \alpha \leq 1; \; p_N \geq 0; \; p_R \geq 0 \]  \hspace{1cm} (6)

### 4 CASE ILLUSTRATION

In this section, the proposed model is illustrated through an example using desktop computers. Suppose that there is an OEM manufacturer conducting both manufacturing and remanufacturing. It is expected that all initial sales will become available for buy-back after four years of use (i.e., \( t = 4 \)), and the company is planning to conduct remanufacturing for the end-of-life products. To maximize the

<table>
<thead>
<tr>
<th>New market</th>
<th>Part worth</th>
<th>Critical value</th>
<th>Competitor 1</th>
<th>Competitor 2</th>
<th>Competitor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU</td>
<td>0.125</td>
<td>(2, 3)</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>RAM</td>
<td>0.125</td>
<td>(2, 3)</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Motherboard</td>
<td>0.100</td>
<td>(2, 3)</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Hard drive</td>
<td>0.050</td>
<td>(3, 5)</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Graphic card</td>
<td>0.025</td>
<td>(3, 5)</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Optical drive</td>
<td>0.050</td>
<td>(3, 3)</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Chassis</td>
<td>0.025</td>
<td>(1, 2)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Selling price</td>
<td>0.500</td>
<td>($1000, $500)</td>
<td>($1000, $500)</td>
<td>($600, $300)</td>
<td>($350, $150)</td>
</tr>
<tr>
<td>Market share</td>
<td>(0.3279, 0.1771)</td>
<td>(0.4906, 0.4861)</td>
<td>(0.1815, 0.3367)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
total lifecycle profit from manufacturing and remanufacturing, the company aims to optimize their product design. To be specific, there are nine product attributes that the company wants to optimize (Table 2), including CPU (central processing unit), RAM (random-access memory), chassis (case, fan, and power supply) and selling price.

The demand for a product is determined by its design (specifications and the selling price) as well as competing product designs. Table 2 shows the target market under consideration in this study. When two numbers are shown in a cell, the former is of the current new-product market and the latter is of the future remanufactured-product market.

In the new-product market, there exist three competing products sold at the prices of $1000, $600, and $350. The current market share indicates that customers prefer most the product with medium specifications (49%) and the highest specifications next (33%). Given the market condition, the expected demand for a new product $D_N$ can be calculated using a conditional multinomial logit choice model, as shown in Equation (7). $Q_N$ denotes the new-product market size, and $U_N$ and $U_j$ denote the customer utility for the new and the competing product $j$, respectively. In the equation, $k$ is a scaling parameter; as $k \to 0$, all choices have the same demand (Jiao and Zhang 2005). Here, $k$ was calibrated on the current market share in Table 2 and defined as 6.45.

The utility for the new product is defined as a linear weighted sum of its generational differences $x_{Ni}$ and the selling price $p_N$. For the calculation, $x_{Ni}$ and $p_N$ are normalized to lie between 0 and 1. The ‘part–worth’ column in Table 2 shows the weight (or part–worth utility) assumed for each normalized $x_{Ni}$ and $p_N$. The ‘critical’ column provides the critical values for $x_{Ni}$ and $p_N$, i.e., $\delta_{Ni}^{max}$ and $\delta_{Ni}^{max} (= p_{Ni}^{max})$. As described in Section 3, the critical values are the maximum generational differences and selling price that customers are willing to accept for a product. In the current study, as an example, the customers will not buy a product if the CPU is more than three generations old.

\[
D_N(t) = Q_N \cdot \frac{\exp(kU_N)}{\exp(kU_N) + \sum_{j \neq N} \exp(kU_j)}
\]

\[
U_N = \sum_{i \neq j} w_{np} x_{Ni}^t + w_{np} p_{Ni}^t \text{ where } x_{Ni}^t = 1 - x_{Ni} / \delta_{Ni}^{max}, \; p_{Ni}^t = 1 - p_N / p_{Ni}^{max}
\]

Equation (9) shows the final demand function obtained for the new product. Similarly, the demand function for the remanufactured product is obtained, where $Q_R$ denotes the size of remanufactured-product market. For simplicity, it was assumed that the part–worth utility does not differ between the new- and the remanufactured-product markets. Also, $k=9.18$ was used for the demand modeling. In this study, $Q_N$ and $Q_R$ are assumed to be 50,000 and 10,000, respectively.

\[
D_N(t) = Q_N \cdot \left(1 + e^{-6.45(U_N - 0.67)}\right)^{-1}
\]

\[
D_R = Q_R \cdot \left(1 + e^{-9.18(U_N - 0.69)}\right)^{-1}
\]

Table 3 provides assumptions on remanufacturing costs and revenues. $V_i^{new}(0)$ represents the market value of the newest cutting-edge part. In Equation (10), it is used for calculating the cost of purchasing a new part. Adopting the model by Kwak and Kim (2011), the equation assumes that a part’s market value depreciates exponentially by its generational difference. The constant parameter $\phi_i$ reflects a part’s own speed of value depreciation. The $\phi_i$ used in this study are given in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>$V_{Ni}^{new}(0)$</th>
<th>$\phi_i$</th>
<th>$\mu_i$</th>
<th>$V_i^{new}$</th>
<th>$r_i(t)$</th>
<th>$\epsilon_i^{second}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU</td>
<td>175</td>
<td>0.6733</td>
<td>0.67</td>
<td>5</td>
<td>0.7745</td>
<td>1</td>
</tr>
<tr>
<td>RAM</td>
<td>50</td>
<td>0.8378</td>
<td>0.50</td>
<td>5</td>
<td>0.7745</td>
<td>1</td>
</tr>
<tr>
<td>Motherboard</td>
<td>150</td>
<td>0.6733</td>
<td>0.67</td>
<td>5</td>
<td>0.5999</td>
<td>1</td>
</tr>
<tr>
<td>Hard drive</td>
<td>120</td>
<td>0.1717</td>
<td>1.00</td>
<td>4.5</td>
<td>0.2787</td>
<td>1</td>
</tr>
<tr>
<td>Graphic Card</td>
<td>100</td>
<td>0.2883</td>
<td>1.00</td>
<td>4.5</td>
<td>0.4646</td>
<td>1</td>
</tr>
<tr>
<td>Optical Drive</td>
<td>80</td>
<td>0.8088</td>
<td>0.40</td>
<td>3</td>
<td>0.0466</td>
<td>1</td>
</tr>
<tr>
<td>Chassis</td>
<td>75</td>
<td>0.1500</td>
<td>0.20</td>
<td>3</td>
<td>0.4646</td>
<td>3</td>
</tr>
</tbody>
</table>
In Table 3, $V_{\text{new}}^{\text{matl}}$ shows the revenue from selling a part to third-party recycler. For simplicity, it is assumed to be the same regardless of the specification. Other processing costs, $c_{\text{takeback}}$ and $c_{\text{market}}$, are assumed to be $58 and $35, respectively (Microsoft, 2008; Bhuie, 2004). Finally, the annual interest rate is assumed to be 3% (i.e., $\theta = 0.03$).

Table 4 shows the optimization result. The results can be summarized as follows:

- The optimal initial design is to include a cutting-edge CPU, RAM, motherboard, hard drive, and chassis, a three-generation-old graphic card, and one-generation-old optical drive. The optimal selling price for the new product is $1000, and the corresponding market share is expected to be 20% (or 10,020 units). The total profit expected from the manufacturing stage is approximately $3.18 million.
- Pursuing remanufacturing can be profitable; it can increase the lifecycle profit by $72,000. To take advantage of the profit opportunity, the company should take back 1,491 units of end-of-life products, which is 14.9% of initial sales. Using the end-of-life products, the company should produce 693 units of remanufactured products. While all other parts are reused in remanufacturing, the RAM and graphic card should be upgraded to cutting-edge and five-generation-old parts, respectively. The optimal selling price of the remanufactured product is $346, and the expected market share is approximately 7%.

5 CONCLUSION

Product design determines both the current profit from the manufacturing stage and the future profit from remanufacturing. To maximize the total lifecycle profit, design decisions must be carefully made considering both stages together. To help in design for lifecycle profit, this paper proposed a mathematical model. The model optimizes both the initial design and design upgrades at the end-of-life stage and also provides corresponding production strategies.

In the future, the model can be improved for multi-objective decision making by incorporating an environmental-impact perspective. Another potentially productive line of research would be to incorporate market trend estimation. The inputs needed for the proposed model (e.g., customer preference, part market value trend, and time-varying reusability) may bring challenges to prediction. Although such future prediction was beyond the scope of this study, a prediction model needs to be developed in the future. Predictive-data mining and time-series analyses (e.g., Ma et al., 2012) may provide a promising solution to this challenge.

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