DETERMINING MODULE REPLACEMENT TIMING OF PRODUCT FOR BALANCING QUALITY AND PRODUCTION COST

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ABSTRACT
As the world has entered the era of limitless competition, most manufacturing companies are trying to reduce the product cost using a modular design strategy. In this environment, the company frequently replaces the subcontractor of modules with another who offers cheaper production cost. Due to insufficient production experience of newly changed subcontractor, adopted module shows higher failure rate than original one. In result, it is important to consider quality cost with production cost. In this paper, we propose a model for determining optimal module replace timing considering both production cost and quality cost through numerical tests.

Keywords: modular design, module replacement timing, product architecture, quality cost, production cost

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1 INTRODUCTION
As the world has entered the era of limitless competition, manufacturing companies are trying to reduce the product cost and lead time. The modular design is one of the most prominent strategies for meeting the manufacturer’s target. It subdivides a product into a number of modules (components), which represent basic units for development or management. By managing each module separately, the modular design provides benefits such as short development lead time and improved maintainability. In a liquid crystal display (LCD) manufacturing industry, modular design is commonly used in most companies. For example, LCD manufacturer is managing modules such as thin film transistor, color filter, back-light, assembly module simultaneously and operating each module (Wang et al., 2007).

In a modular design, the company often subcontracts the production of each module to other companies. In this situation, it is common that the company frequently replace the subcontractor with others who offers cheaper production cost (Chopra et al., 2007). For example, unit price of module is able to change from 10 dollars to 8 dollars (20% reduction) by changing company “B” to “A”, or 10 dollars to 6 dollars (40% reduction) by changing factory location from Europe to Asia. This module replacement could happen at several times a year.

However, module change accompanies potential risk in quality management. Due to the lack of production experience, the newly adapted subcontractor produce module with relatively lower quality. (Yun and Choi, 2000) reveals the fact that the newly adapted module shows higher failure rate than original module.

Therefore, it is important to decide module replacement timing because it could affect both production cost and quality cost. When a company replaces a low-cost module late, it negatively affects the production cost. On the other hand, when a company adopts a low-cost module early, the company could suffer quality issues due to insufficient experience of subcontractors. In this sense, there is a trade-off between quality cost and production cost in module replacement timing. In this situation, the company must decide whether to replace the existing modules with the low-cost module for cost reduction or not.

In this study, we propose a model for determining optimal module replace timing considering both production cost and quality cost. Our study determines an optimal timing of multi-module replacement when several modules interact with each other. Because of the interdependency between modules, product architecture is used as an input to the model. In addition, this paper confirms that some factors (cost discount rate, technology level which is related to quality) have an effect on replacement timing through numerical test. Previous papers which are concerning optimal timing decision are mainly related to find the facility replacement or product replacement timing in terms of repair cost and maintenance aspects in manufacturing company. However this paper suggests the modeling which calculates quality and production cost after application of new modules, and recommends optimal timing of module replacement.

The remainder of this paper is organized as follows. Section 2 reviews related literature for calculation of quality and production cost. Then we present the basic modeling frame work and the assumption of our model in Section 3. And, we present an extensive numerical experiment in Section 4. And in Section 5 we describe the experiment results and discussions. A summary of our findings and directions are presented in Section 6.

2 LITERATURE REVIEW

2.1 Product replacement timing
Product replacement problems have long been studied in the reliability research literature (Barlow et al., 1965). Product replacement timing means to determine when to replace the product considering such as repair, maintenance, quality of product (Yun and Choi, 2000). In order to find the optimal replacement timing, Aven and Dekker (1997) presents the framework of optimal replacement model by considering marginal cost. Hopp and Nair (1991) studies decision time to replace by considering the discontinuous technology change. John and Robert (1994) presents heuristics in replacement timing of durable products. Dandy and Engelhardt (2001) presents the optimal scheduling of water pipe replacement using genetic algorithms.
In addition, there are some studies focusing on the maintenance problem. For example, Yun and Choi (2000) studies optimal replacement intervals which minimize the repair and maintenance cost with random time horizon. Liu and Huang (2010) present the optimal replacement policy for multi-state system when a failure occurs.

In general these studies tend to find the optimal replacement time of the facility or the product by minimizing repair and maintenance cost in terms of the reliability of the product. Our study is distinguished from others by focusing on module replacement timing with the consideration of product architecture. In recent years most manufacturing companies tend to reduce the product price using replacement of subcontractor who suggests cheaper price due to limitless competition. Therefore, finding optimal timing for module replacement is an important issue because it could affect the overall cost over the life cycle of the product.

2.2 Failure rate

Failure rate is the frequency with which an engineered system or component fails. The failure rate of a product usually depends on time, with the rate varying over the life cycle of the system. In reliability theory, the bathtub curve is widely used in order to describe pattern of failure rate. According to bathtub curve, there are three phases of failure rate - first phase when failure rate of system is sharply decreasing, second phase when failure rate of system is constant, and third phase when failure rate of system is sharply increasing (Urban, 1980).

In this paper, we assume that the failure rate of the module has decreasing failure rate (DFR) properties. The DFR distribution satisfies that the conditional survival probability which is not failure at some stage is steadily increasing as a function of stage (Barlow et al., 1975). In order to explain and estimate the failure rate which is followed DFR, gamma distribution is typically applied. According to Gort and Klepper (1982), the failure rate should be modeled in concerned with product life cycle stages because the behavior of the product reliability is different at each stage. In the early stage when the specification of product is frequently changed, technology and market related to product has high uncertainty. Therefore what is important in managing early stage is how to satisfy the customer needs by proper technology or technological solutions (Jovanovic and MacDonald, 1994a; 1994b). On the other hand, in the later stages, specification and needs of product is completely determined, hence technology and market have low uncertainty.

Therefore, the uncertainty lies in each product module differs according to their product life cycle. The degree of technological activity is applied to assign the product whether high-tech or low-tech (Nelson and Winter, 1978; Audretsch, 1995). In this paper, each module of product is distinguished between high-tech and low-tech based on the degree of technological uncertainties. In the study of Rajshree (1997), high-tech and low-tech products are compared by using the time-series data which consists of product failure rate of several markets. As a result, there are two meaningful results of analysis with estimated failure rate. First, in the early phase, the failure rate of high-tech product is higher than that of low-tech product. Second, both high-tech product and low-tech product have a DFR distribution which means failure rate is steadily decreasing as time passed. Based on this literature background, we apply that the degree of technology development affects the failure rate in this paper.

2.3 Product architecture

In general, a product consists of modules which include several components. Typically, the certain module has complicated interactions across other modules simultaneously. In an attempt to better express module interactions, design structure matrix (DSM) has been used at product architecture (Browing, 2001). The purpose of DSM is simply to describe a function interactions and performance effect across modules by matrix. In order to estimate the comprehensive interaction between modules, it is necessary to verify direct and indirect relation across modules. One of the most widely used methodology in estimating such interaction is Clarkson et al. (2004)’s change prediction method (CPM) shown in the Figure 4. For example, $l_{a,b}$ could be updated $L_{a,b}$ by CPM which is including direct and indirect relations. Then the updated matrix could consider the overall effect across modules. In this study, we apply the same logic in estimating a failure rate of each module considering interactions between other modules.
3 PROPOSED APPROACH

3.1 Trade-offs between failure rate and production cost
When a low-cost module is available in the market, companies must decide whether to replace the existing modules with the low-cost module. The replacement of existing modules might be effective in terms of reducing the overall production cost. However, the failure rate of the modules can be rising when new modules are aggregated. In addition, it is necessary to consider aspects of the production cost and failure rate depending on the time until the production cost and failure rate are a changing function of time.

The trade-offs between the failure rate and the production cost in accordance with the replacement of the modules is shown in Figure 3. If the module is replaced, the failure rate $f(t)$ increases sharply from existing stable level because the stability of new module is lower than that of existing module. However, the production cost $C(t)$ decreases because the producer of the new module might offer lower price.

![Figure 3. (a) Failure rate trend (b) Production cost trend by replacement at time x](image)

3.2 Assumptions and input variable

*Figure 4* shows our research framework according to our modeling phase. In this paper, we modeled the discontinuous time-specific cost changes due to module replacement by considering the quality cost according to the failure rate and production cost.

![Figure 4. Research Framework](image)

For the cost modeling, we assume the following model specifications. First, the product architecture does not change over time. In other words, we assume that the change of failure rate according to the
degree of interaction is constant because of the relations between the modules are kept constant. Second, the failure rate of the new module follows decreasing failure rate (DFR) function which gradually decreases over time, but the failure rate of existing module is kept constant because they are in a stable status. Third, quality cost is calculated as the reproduction cost caused by the failure of the module and the production cost is gradually decreasing depending on the time. Fourth, the cost of the new module is always cheaper than an existing module. Finally, we assume that the production unit of modules remains constant.

In addition, input variables should be defined in order to model the failure rate when modules are changed. For cost modeling it is needed for the inter-module DSM data of products as input variables in Figure 5. In this DSM matrix, the initial failure rate and the degree of inter-module relations are listed. For example, a failure rate of the module A is 0.07 and relation from B and A is 0.22 in Figure 5.

![Figure 5. Initial DSM and Changeable Modules](image)

Finally, the modules which are arranged to decreasing of production cost should be selected. In addition, the cost discount rate by module change, the cost reduction rate of modules and the discount rate are arranged initial values, and these values are described in details for each modeling step.

In this paper, in order to determine replacement timing of modules, we replace the modules each time and compare the total cost which is calculation by quality and production cost of each period. Because the failure rate and production cost are measured differently before and after replacement of modules, we propose the each modeling existing modules and replacement modules. We describe each modeling in details in the next section.

### 3.3 Cost modeling

#### 3.3.1 Failure rate depending on time

In this study, we have two assumptions for modeling failure rate of the module. First, failure rate of all modules decreases steadily as time passed until the stabilized phase. Second, since the existing module is already in the stabilized phase, the failure rate of the existing module remains constant. In order to model the failure rate of the module over time, we use the gamma distribution which is widely used in reliability theory. The probability density function of gamma distribution is defined in equation (1), and the gamma function is defined in equation (2).

\[
g(w) = \frac{\lambda^\alpha}{\Gamma(\alpha)} w^{\alpha-1} e^{-\lambda w} \tag{1}
\]

\[
\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx (\alpha > 0) \tag{2}
\]

In equation (1), \( \lambda \) is a scale parameter which is strictly positive and \( \alpha \) is a shape parameter which reflects the characteristic of module in viewpoint of reliability. Also, if \( \alpha \) is determined, the gamma function becomes constant.

By the monotonicity properties of the gamma function, the function of failure rate \( f(t) \) can be derived as equation (3) (Richard, 1975).

\[
(f(t))^{-1} = \frac{1-G_{\alpha,\lambda}(t)}{g_{\alpha,\lambda}(t)} = \int_0^{\alpha} \left( \frac{x}{t} \right)^{\alpha-1} e^{-\lambda(x/t)} dx \tag{3}
\]

In equation (3), \( G(t) \) is cumulative distribution function of gamma distribution and \( g(t) \) is probability density function of gamma distribution. When \( \alpha \) is larger than 1, the failure rate has decreasing failure rate (DFR) properties. In addition, development degree of technology is also considered to
estimate the function of the failure rate, because the initial failure rate and the rate of change is different whether in the high-tech or low-tech module. This degree of technology is reflected by the value of \( \alpha \) which is shape parameter in gamma distribution. The value of \( \alpha \) in the high-tech module is higher than that in the low-tech module.

### 3.3.2 Updated failure rate

In order to find the updated failure rate of each module, we adopted the Clarkson et al. (2004)’s method for calculating both direct and indirect propagation effect between modules. The updated failure rate of each module is calculated in equation (4)

\[
F_i = f_i + \sum_{k=1}^{n} L_{ik}
\]

where \( L_{ik} \) is the amplified effect on module \( i \) caused by failure rate of module \( k \), and \( f_i \) is the initial failure rate of module \( i \). Using the equation (4), the updated failure rate of each module can be calculated for each period.

### 3.3.3 Production cost and Quality cost

In this study, cost consists of two parts, production cost and quality cost. The production cost of each module can be calculated by equation (5)

\[
P_i(x) = \sum_{i=1}^{N} \left( C_{i,e}(t) \times \frac{1}{(1+r)^n} \right) + \sum_{i=1}^{N} \left( C_{i,n}(t) \times \frac{1}{(1+r)^n} \right)
\]

where \( C_{i,e}(t) \) is the unit cost of existing module \( i \) and \( C_{i,n}(t) \) is the unit cost of new module \( i \). The decision variable \( x \) is the module replacement timing. Therefore the left summation in equation (5) means accumulated production cost using existing module before replacement, and right summation means accumulated cost after module replacement. The total number of product \( N \) is determined a priori. The discount rate \( r \) is used for calculating net present values of total cost.

In a similar way, the quality cost of each module can be calculated by equation (6).

\[
Q_i(x) = \sum_{i=1}^{N} \left( f_{i,e}(t) \times N \times C_{i,e}(t) \times \frac{1}{(1+r)^n} \right) + \sum_{i=1}^{N} \left( f_{i,n}(t) \times N \times C_{i,n}(t) \times \frac{1}{(1+r)^n} \right)
\]

Unlike the production cost, quality cost also considers the failure rate of the module at stage \( f_i(t) \). Therefore we introduce \( f_{i,e}(t) \) and \( f_{i,n}(t) \) as failure rate of either existing module \( i \) and new module at time \( t \).

The total cost \( T_i(x) \) is calculated by equation (7).

\[
T_i(x) = Q_i(x) + P_i(x)
\]

Therefore, our object determine optimal \( x \) which minimizes the total cost.

### 4 NUMERICAL EXAMPLE

In this section we conduct a numerical experiment for the illustration of our approaches. First, we assume that the product is composed of 4 modules (A, B, C, and D) and product production quantity is constant as 50,000 units. The initial failure rate and relations between modules are defined in Figure 5 in section 3.2. The changeable modules are assigned in A and C, and for calculation failure rate, the gamma function parameter \( \alpha \) is determined according to their technology levels. According to section 3.3.2’s update failure rate modeling, each updated failure rate is determined in each period. The initial production cost is determined by the unit price ($) of the module and the price of module A, B, C, and D is 15,000, 15,000, 15,000 and 13,000 each. And the cost discount rate by module change is set to be changed. And we set the cost reduction rate of module in times is 5% and the discount rate is 3% likely interest rate. Also, because we propose the optimal replacement timing according to the failure rate and the production cost, we define other variables are constant except for these two variables.

In order to examine the difference of optimal replacement timing depending on 1) the cost discount rate by module change and 2) the initial failure rate by technology level, we composed 3 experimental sets as follow (Table 1).

By the above 3 kind of experimental sets, we perform experiments by replacing the changeable modules in each of the four periods. And, under the same conditions except the degree of price reduction and the initial failure rate, we observe the optimal replacement timing in each case and the cause of these differences.
<table>
<thead>
<tr>
<th>Experimental Set</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 Technology Level</td>
<td>A : High ( $\alpha =0.9$), C=Low( $\alpha =0.2$)</td>
<td>A : 10%, C : 10%</td>
<td>A : 15%, C : 15%</td>
</tr>
<tr>
<td>Cost Reduction Rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2 Technology Level</td>
<td>A, C : High ( $\alpha =0.9$)</td>
<td>A : 10%, C : 10%</td>
<td>A : 15%, C : 15%</td>
</tr>
<tr>
<td>Cost Reduction Rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3 Technology Level</td>
<td>A,C : Low( $\alpha =0.2$)</td>
<td>A : 10%, C : 10%</td>
<td>A : 15%, C : 15%</td>
</tr>
<tr>
<td>Cost Reduction Rate</td>
<td></td>
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</table>

5 RESULTS AND DISCUSSIONS

5.1 Experiment 1: Cost discount change

According to the experiment setting in section 4, we derive the results and discussions. In this experiment setting (#1), we observed the difference of optimal replacement timing of modules according to the degree of the cost discount rate. We conduct experiment increasing the cost discount rate of changeable modules (A and C) such as 10%, 15%, and 20%, but the other variables are maintained constantly. Through this experiment we check the difference of optimal replacement timing according to the cost discount rate changing.

![Figure 6. Results of experiment 1(left: case 1, middle: case 2, right: case 3)](image)

The experiment results depending on the replacement of modules (A and C) in each period are as shown in Figure 6. According to the results in Figure 6, the optimal replacement timing appears to be the stage 2, stage 1, and stage 0 when each cost discount rate (10%, 15%, and 20%). These results show that 1) when the cost discount rate is low, changing modules in late period which modules are in a stable status, is better than changing in early period and 2) when the cost discount rate is high, changing modules in initial period is effective. This result is reasonable because the larger cost discount rate, effect of production cost on total cost is larger and the smaller cost discount rate, effect of quality cost on total cost is larger. Therefore, this result shows that there are tradeoff between the quality and production cost.

5.2 Experiment 2: Low technology modules

In this experiment setting (#2), we observed the difference of optimal replacement timing of low-tech modules according to the degree of the cost discount rate. We set the changeable module (A and C)’s $\alpha$ as 0.2 and conduct experiments in increasing the cost discount rate of changeable modules such as 10%, 15%, and 20%, but the other variables are maintained constantly. Through this experiment we checked the difference of optimal replacement timing when the changeable modules are low technology modules.

The experiment results depending on the replacement of modules (A and C) in each period are as shown in Figure 7. According to the results in Figure 7, in case of the low technology modules, the optimal replacement timing appears to be the stage 0 in all experiments. This result shown that effect of product cost on total cost is larger than quality cost because the low technology modules have low...
failure rate according to equation (3) in section 3.3.1. Therefore, this result means if the lower production cost modules are presented in a market, replacing modules immediately is more efficient in case of low technology modules.

5.3 Experiment 3: High technology modules

In this experiment setting (#3), we observed difference of the optimal replacement timing of high-tech modules according to the degree of the cost discount rate. We set the changeable module (A and C)’s \( \alpha \) as 0.9 and conduct simulations increasing the cost discount rate of changeable modules such as 10%, 15%, and 20%, but the other variables are maintained constantly. Through this experiment we check the difference of optimal replacement timing when the changeable modules are high technology modules.

The experiment results depending on the replacement of modules (A and C) in each period are as shown in Figure 8. According to the results in Figure 8, in case of the high technology modules, the optimal replacement timing appears to be stage 2, stage 1, and stage 1 when each cost discount rate 10%, 15%, and 20%. This result shown that effect of quality cost on total cost is larger than production cost because the high technology modules have high failure rate according to equation (3) in section 3.3.2. Therefore, this result means if the lower production cost modules are presented in market, replacing modules in a stable period is more efficient in case of high technology modules.

6 CONCLUSIONS

This paper discusses how to select optimal replacement timing for module changes from failure rate and production cost point of view. The previous research projects which are concerning optimal timing decision are mainly related to find the facility replacement or product replacement timing for repair cost and maintenance. So, this paper provides a new framework of making an optimal timing decision of module replacement considering not only production cost but also quality cost. For this cost modeling, we have considered several features arising in the context of module replacement. First we have defined the tradeoff between failure rate and production cost as to select optimal replacement timing. Because of this tradeoff there is optimal replacement timing in changing modules. And the failure rate of each module for calculating quality cost affected by technology levels.
of modules and timing of their lifecycle. To consider quality cost, we estimate failure rate value by assuming DFR of gamma distribution with technology level. Finally for calculating the holistic failure rate of product, we use change propagation methods according to relations of modules. Based on the updated failure rate, we calculate the total cost and make a timing decision for module replacement. For verifying our proposed approach, we perform an experiment which consists of three experiment sets. And through this experiment, we identify our approach is efficient for selecting the optimal replacement timing of modules.

Our research can be used in manufacturing companies to determine the optimal replacement timing of modules and based on this method, manufacturing companies can benefit by reducing the cost. And this approach also can present the replacement timing depending on the technology levels and the price of modules.

As a final comment, we should mention the limitations of our research and potential future works. This paper assumes that replacement timing is considered by discrete time point of view. And failure rate is following DFR trend. But in reality, replacement timing will be continuous and failure rate also follows IFR (Increasing Failure Rate) trend when modules become superannuated. So, because of these limitations, our research can expand discrete timing decision to continuous timing decision and consider the failure rate which is following IFR trend as well. In additions, this subject can be expanded from replacement timing decision to optimal combination of replacement module at certain timing.

ACKNOWLEDGMENTS
This work was supported by the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea Government (MEST) (No. 0415-20120039)

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