# APPROACHES FOR MAPPING BETWEEN PREFERENTIAL PROBABILITIES AND RELATIVE DESIGN PREFERENCE RATINGS 

Haifeng JI, Tomonori HONDA, Maria C. YANG<br>Massachusetts Institute of Technology, United States of America


#### Abstract

Assigning preferences to a set of design choices is an important activity in the design process. Previous research proposed a probabilistic approach to extracting preference information from transcripts of design team discussion in a low overhead, implicit way. However, the preference information that was extracted took the form of a "preferential probability," rather than a more traditional preference rating. Preference ratings describe the strength of how much a design team prefers a design choice, and several formal design techniques require such preference ratings. This paper examines the underlying theoretical mappings between preferential probabilities and relative preference ratings, and explores the feasibility of converting preferential probabilities into to relative preference ratings. The paper presents an algorithm for performing this conversion, and then illustrates the use of the algorithm by applying it to a case example. The method proposed in this research has the potential to link implicit preference information generated by real world design teams with formal design decision-making tools.


Keywords: design process, decision making, conceptual design, design preference
Contact:
Dr. Haifeng Ji
Massachusetts Institute of Technology
Engineering Systems Division
San Jose
95132
United States of America
haifengj@mit.edu

## 1 INTRODUCTION

In the formulation stage of engineering design, teams clarify design problems, generate possible solutions, and select appropriate candidates for further investigation. This selection may be thought of as a process of assigning priorities, or preferences, to the set of possible design choices. This assignment process can be challenging when conducted by a team rather than by an individual. Imagine that a design team discusses alternatives for a design in order to select one. Individuals will have their own preferences, though they may not always express them explicitly. These preferences may also change over the course of discussion. Challenges include the accurate elicitation of preferences, appropriately balancing the trade-offs between various design choices, and effectively aggregating individual team member preferences into a single group preference.
To address these issues, previous work (Ji et al. 2007, Ji et al. 2012) offered an approach to extracting implicit preference information embedded in the transcripts of a team's deliberations, known as Preferential Probabilities from Transcripts (PPT). The approach has been extended to include formal linguistic appraisal (Honda et al. 2010), a time-based preference Markov model based on design linguistics (Dong et al. 2011), and a sliding-window approach for finer granularity for implicit preferences (Ji et al. 2011). These approaches draw on the discussion among team members to determine a pattern of preference-related information over time. It is a descriptive approach that avoids aggregating group information and instead treats the entire team's discussion as a single set of words. In all of these approaches, preference information takes the form of a preferential probability rather than a traditional numeric preference rating. A preferential probability of an alternative is the likelihood it will be preferred over all others, also known as the "most preferred" probability.
Existing formal tools for design decision-making such as Quality Function Deployment (Hauser and Clausing 1988) and the Method of Imprecision (Wood and Antonsson 1989, Otto and Antonsson 1993) require the input of a numeric preference rating. The research question this paper asks is: How can preferential probabilities be effectively converted into numerical preference ratings? The ability to convert preferential probabilities into preference ratings would allow us to make the powerful link between the implied decisions made during free-form, natural language team discussion directly to formal, structured design methods. This paper details the conditions for mapping between preferential probabilities and preference ratings, then describes previous work translating ratings into these probabilities, and finally presents a new approach for mapping probabilities into relative ratings. Note that "relative" preference ratings in this paper are constrained to sum to 1 for the set of preference ratings. This approach to mapping relative numeric preference ratings and preferential probabilities will establish a way to link informal team discussion about design alternatives to sophisticated formal decision-making tools in a way that has not been possible before.

## 2 RELATED RESEARCH

Design decision-making. Engineering design research has focused on a number of methodologies for making design decisions, including Pugh charts (Pugh 1991), Quality Function Deployment (Hauser and Clausing 1988), and Decision-based Design (Hazelrigg 1998). A critical phase in employing these methodologies in design selection is to consider multiple criteria, and to make trade-offs when criteria are at odds. Utility theory (von Neumann and Morgenstern 1947, Tribus 1969, Keeney and Raiffa 1976, Howard and Matheson 1984, Thurston 1991) and the Method of Imprecision (Wood and Antonsson 1989, Otto and Antonsson 1991) are well known methods for formalizing trade-offs.
Design preferences. The determination of preferences has been approached in a number of ways. The lottery method (Krantz et al. 1971) scales alternatives from 0 to 1 . Approaches for eliciting preferences through pair-wise comparisons include the Analytical Hierarchy Process (Saaty 2000) and the fuzzy outranking method (Wang 1997). In multi-attribute design, aggregation functions (Scott and Antonsson 1998) can be applied to calculate overall preferences. Methods drawn from factorial design and statistics can be used to elicit preferences, including conjoint analysis (Green and Srinivasan 1990) and discrete choice analysis (Hensher and Johnson 1981, Ben-Akiva and Lerman 1985).
These methods either make assumptions about preference values, or are stated value methods that require explicit elicitation from stakeholders. Preferential Probabilities from Transcripts (Ji et al. 2007) is an implicit strategy for extracting preference-related information in probabilistic form from design team transcripts.

Preferences of a team. There is a rich literature on group decision-making styles and qualitative ways to aggregate team opinion (Vroom and Jago 1988). Studies on decision-making in engineering design have centered around more structured methods for aggregation. Arrow's Impossibility Theorem (Arrow 1970, Arrow and Raynaud 1986) posits that there is no guarantee of consistency for aggregation of preferences in a group. Nevertheless, research has considered strategies for making sense of individual ratings. Dym et al. (2002) have described a structured pairwise comparison chart for aggregating individuals' preference ordering. Keeney makes the assumption that each group member's opinion is equally important and uses cardinal utility functions to accumulate group preferences (Keeney 1976). Jabeur et al. (2004) and See and Lewis (2006) have considered unequal weights on the preferences of the group members. In our work on PPT (Ji et al. 2007, Honda et al. 2010), no knowledge of an individual's preferences is required, nor are individual preferences aggregated. The group is instead thought of as a single entity that generates information about the group's overall preferences throughout discussion. Group preferences are extracted from transcripts of team discussion, without any aggregation. Honda et al. (2010) applied formal linguistic appraisal to the method to identify a group's attitudes towards design alternatives and extract corresponding preferential probabilities. Dong et al. (2011) formulated a time-based Markov model to asses team preferences using formal linguistics.
In several of these studies, the baselines for comparison are surveys of the preferences of designers. A method for deriving preferential probabilities from surveys (PPS) (Ji et al. 2012) was developed to translate ratings and rankings into preferential probabilities. However, only conversion from ratings to preferential probabilities was explored. These extracted preferential probabilities cannot be used directly as inputs to formal design methods. This study proposes an algorithm which can convert preferential probabilities into the relative group preference ratings.

## 3 METHODS

### 3.1 Relationship between Preferential Probabilities and Relative Preference Ratings

The key challenge in mapping between preferential probabilities and preference ratings lies in the distribution of preference values. Previous work (Honda et al. 2010, Ji et al. 2012) assumed that preference ratings exist in a bounded distribution with the expected value of the stated ratings. Using the principle of maximum entropy (Jaynes 1957, Jaynes 1968), which provides the least biased distribution for the given information, the joint distribution of ratings is described as in Equation (1).

$$
f\left(x_{1}, x_{2} \ldots, x_{N-1}\right)=\left\{\begin{array}{cc}
\lambda_{0} e^{\lambda_{1} x_{1}+\lambda_{2} x_{2}+\ldots+\lambda_{N-1} x_{N-1}}, & \text { if } 1-u_{N} \leq \sum_{k=1}^{N-1} x_{k} \leq 1-l_{N} \text { and } l_{\mathrm{i}} \leq x_{i} \leq u_{i} \forall i \in[1, N-1]  \tag{1}\\
0, & \text { otherwise }
\end{array}\right.
$$

Where $x_{1}, x_{2}, \ldots x_{N-1}$ are the sampling variables for the relative ratings for Alternative $1, \ldots, N-1$. Sample variable $x_{N}$ for Alternative $N$ is implicated by the constraint that $x_{N}=1-x_{1}-x_{2^{-}} \ldots-x_{N-1} .\left[l_{i}, u_{i}\right]$ is the range of the relative values for Alternative $i(1 \leq i \leq N)$. The above distribution function shows that the joint exponential distribution is only meaningful when all the sample variables are in the possible ranges, otherwise it is zero.
Parameters $\lambda_{0}, \lambda_{1}, \ldots \lambda_{N-1}$ can be solved from the following $N$ equations.

$$
\begin{align*}
& \int_{l_{1}}^{u_{1}} \int_{l_{2}}^{u_{2}} \ldots \int_{l_{N-1}}^{u_{N-1}} f\left(x_{1}, x_{2} \ldots, x_{N-1}\right)=1 \\
& \int_{l_{1}}^{u_{1}} \int_{l_{2}}^{u_{2}} \ldots \int_{l_{N-1}}^{u_{N-1}} x_{1} f\left(x_{1}, x_{2} \ldots, x_{N-1}\right)=r_{1}  \tag{2}\\
& \int_{l_{1}}^{u_{1}} \int_{l_{2}}^{u_{2}} \ldots \int_{l_{N-1}}^{u_{N-1}} x_{N-1} f\left(x_{1}, x_{2} \ldots, x_{N-1}\right)=r_{N-1}
\end{align*}
$$

Where $r_{1}, r_{2}, \ldots r_{N}$ are designers' stated relative ratings which are the expected value of the variables from the distribution. The first equation guarantees that the total probability is 1 integrated over the possible rating ranges for the joint distribution, and the next $N-1$ equations set the requirements for the expected values for variable $x_{1}$ to $x_{N-1}$. The expected value for variable $x_{N}$ is met tacitly because $E\left(x_{N}\right)=E\left(1-x_{1}-x_{2}-\ldots x_{N-1}\right)=1-r_{1}-r_{2}-\ldots r_{N-1}=r_{N}$.
Monotonicity: Without loss of generality, we assume $r_{k}(k=2, \ldots N-1)$ is kept constant. When the stated rating $r_{I}$ increases to its upper boundary, the parameter $\lambda_{I}$ also increases to fit the distribution. When $\lambda_{I}$ increases, there will be more sample ratings for Alternative 1 near the upper boundary, and the probability that it is greater than the samples from other alternatives will increase, which indicates higher preferential probabilities for Alternative 1. This indicates monotonicity between relative preference ratings and preferential probabilities.
Continuity: Continuity can be proved under the theorem that all elementary functions are continuous within a range. Without loss of generality, the function in Equation (1) is an elementary function on $\lambda_{1}$, so it is continuous on the range $(-\infty, \infty)$ for $\lambda_{1}$, and the integral of f on $x_{1}, x_{2}, \ldots x_{N-1}$ is still continuous in the range $(-\infty, \infty)$ for $\lambda_{1}$. Both preferential probabilities and relative ratings have an integral relationship with $\lambda_{1}$. In this way, the preferential probability of Alternative 1 is continuous with $\lambda_{1}$, and the relative rating of Alternative 1 is continuous with $\lambda_{l}$; and vice versa. Therefore, the preferential probability of Alternative $l$ is continuous with the relative rating of Alternative 1 .
Bijection: Let $r_{i}(1 \leq i \leq N)$ be the relative preference rating for Alternative $i$ and $p_{i}(1 \leq i \leq N)$ be the preferential probability for Alternative $i$ in a design selection problem with $N$ alternatives. A mapping function can be described as
$F:\left(r_{1}, r_{2}, \ldots r_{N}\right) \rightarrow\left(p_{1}, p_{2}, \ldots p_{N}\right)$, where $r_{i} \geq 0(1 \leq i \leq N)$ and $r_{1}+r_{2}+\ldots+r_{N}=1$.
Due to the constraint that $r_{1}+r_{2}+\ldots+r_{N}=1$ and $p_{1}+p_{2}+\ldots+p_{N}=1$, this mapping function can be reduced to ( $N-1$ ) dimensions:
$F:\left(r_{1}, r_{2}, \ldots r_{N-1}\right) \rightarrow\left(p_{1}, p_{2}, \ldots p_{N-1}\right)$, where $r_{i} \geq 0(1 \leq i \leq N)$ and $r_{1}+r_{2}+\ldots+r_{N-1} \leq 1$.
In the given range, the mapping is continuous and monotonically increasing. So for any ( $N-1$ )dimensional vector of ratings, there is one and only one vector of preferential probabilities, and for any vector of preferential probabilities, there is one and only one corresponding relative rating. In other words, this function is bijective and maintains a 1-to-1 mapping.
Symmetry: From Equation (1) and Equations (2) to ( $\mathrm{N}+1$ ), we can deduce that for any mapping,
$F:\left(r_{1}, r_{2}, \ldots r_{N}\right) \rightarrow\left(p_{1}, p_{2}, \ldots p_{N}\right)$, if the values for $r_{m}$ and $r_{n}(1 \leq m, n \leq N)$ are exchanged while the other ratings are kept unchanged, then the mapping still holds with only the values of $p_{m}$ and $p_{n}$ exchanged. As the mapping is a bijective 1-to-1 mapping, the inverse mapping,
$F^{\prime}: \rightarrow\left(p_{1}, p_{2}, \ldots p_{N}\right) \rightarrow\left(r_{1}, r_{2}, \ldots r_{N}\right)$, has the same property.

### 3.2 Forward Mapping: Relative Preference Ratings to Preferential Probabilities

Section 3.1 describes the conditions of the relationship between preferential probabilities and relative numeric preference ratings. In this section, we consider one way to map between them. A method called Preferential Probabilities estimated from Surveys (PPS) (Ji et al. 2012) can be applied to translate ratings into preferential probabilities. This approach is drawn from the principle of maximum entropy (Jaynes 1957, Jaynes 1968) and it does not assume a deterministic distribution a priori for preference ratings. The distribution and parameters are calculated while maximizing information entropy so that it does not have any unknown parameters.
Details of the derivation of PPS are given in (Ji et al. 2012). Rather than specifying a number of points (e.g. 10 points) to allot among all alternatives, we set the sum of all alternative ratings to 1 because ratings are all relative in this study. The steps for PPS for converting group relative preference ratings to preferential probabilities are summarized as follows:

1. Construct a joint distribution for group's ratings for all alternatives, as described in Equation 1;
2. Randomly select a sample as the "true" group rating based on the joint distribution. The rejection method (Ross 2006, Press et al. 2007) can be employed to generate samples;
3. Compare the group ratings to determine the most-preferred alternative;
4. Repeat Steps 1-3 until the predefined number of simulation runs is reached. The group's preferential probability for an alternative can be estimated from statistical simulation by calculating the proportion of runs when this alternative has the highest simulated group rating. In the case study in this paper, 10,000 simulation runs were used for statistically estimating the probability values.

### 3.3 Inverse Mapping: Preferential Probabilities to Relative Preference Ratings

Finally, this paper describes a strategy for mapping preferential probabilities into relative preference ratings. The challenge is that simulation-based approaches are difficult to control. It should also be noted that the resulting ratings are not deterministic values but the expected preference values with maximum likelihood. It is not trivial to describe the mathematical relationship from the preferential probabilities as explanatory variables to the relative ratings as response variables, based on the known mathematical equations in Section 3.1. Response Surface Methodology (RSM) (Box and Wilson 1951) is a statistical technique for exploring the relationship between explanatory variables and response variables when the model or the parameters are unclear. Although the main idea of RSM was for optimal response with experiment design, it could also be used to estimate the approximate response variables (e.g. relative ratings) from any given set of explanatory variables (e.g. preferential probabilities).
For generating the experimental data for statistics analysis, we could apply the algorithm in Section 3.2 to generate sample sets of preferential probabilities with given sets of relative ratings. One difficulty associated with formulating a numerical approximated response function is that samples of preferential probabilities are determined from simulation results of forward mapping, and it is very difficult to control the location of the samples of preferential probabilities. Thus, it is necessary to assume that samples are not located in a regular rectangular grid. Common techniques like polynomial interpolation, spline interpolation and linear interpolation require a rectangular grid. An approach that works well for this irregular domain is Akima interpolation (Akima 1978). An appealing property of this technique is that it guarantees continuity and local differentiability that a mapping from a probability to a rating should have.
The following steps collect relative preference ratings by inverse mapping preferential probabilities:

1. Divide the space for relative preference ratings into suitably small ranges; this scale of the range can be determined by the tradeoff between the simulation time and the tolerance errors.
2. Increase preference ratings from the lower bound for each alternative by the size chosen in Step 1 , and generate the sample for the combinations of ratings for all alternatives. For example, when $1 / 12$ is chosen as the small range for a 3-alternative design selection, there are 19 combinations, such as $(0,0,1),(0,1 / 12,11 / 12),(0,1 / 6,5 / 6)$, etc. Due to the symmetry of preferential probabilities and relative preference ratings in the mapping discussed in Section 3.1, we can minimize the number of the samples by employing the constraint $r_{1} \leq r_{2} \leq \ldots \leq r_{N}$.
3. Use the forward mapping approach described in Section 3.2 to obtain the preferential probabilities for the sampled relative preference ratings.
4. Use permutation (switching the order of the sampled preferential probabilities and the order of the corresponding relative preference ratings) to build the statistical mapping between the vector of preferential probabilities (explanatory variables) and the relative ratings of the first alternative (response variables). Permutation is utilized to reduce dimensionality of the response. Rather than fitting to produce interpolation from N -Dimension vector of preferential probabilities to N Dimension vector of relative preference ratings, we are utilizing permutation to just do N Dimension to 1 -Dimension mapping and simplifying the surface fitting process.
5. Use Akima interpolation based on the existing statistical sample data to obtain the relative rating of the first alternative for any given vector of preferential probabilities.
6. Relative ratings for other alternatives can be found by reapplying the permutation of the given preferential probabilities on the statistical data. For example, we can switch the first preferential probability for the second and run Akima interpolation to find the relative rating for the second alternative.
With the approaches for forward- and inverse-mapping, these two forms of preferences are interconvertible, and the conversion between them bridges the gap between the implicit preferences extracted probabilistically and the established decision-making framework using preference ratings.

## 4 CASE STUDY

### 4.1 Case Description

The data source for this case is a transcript of a discussion about design selection by a team of three engineering graduate students at a US university. One had 7 years of work experience, one had 2 years, and the third had none.

Before the experiment, each team member was given a think-aloud training exercise to practice naming each alternative with proper names rather than ambiguous pronouns ("this" or "that") to enable the tracking of design alternatives in the transcript. For example, a "plastic carafe" had to be called "plastic carafe" or "plastic pot". During the experiment, the team members were asked to reach consensus through discussion, and they were also asked to individually complete written surveys every $\sim 10$ minutes with their preference ratings for design choices. The experiment lasted 50 minutes, including 10 minutes for instruction and training, and 8 minutes for 5 surveys.
The team's task was to consider two design selection problems, for a carafe and a filter for a coffeemaker. Each had 3 candidate alternatives defined beforehand. Total cost for these 2 components could not exceed $\$ 35$, and the intended user was a retired person who is a coffee connoisseur.
Preferences may be ambiguous. Hey (1998) and Kulok and Lewis (2005) found that human designers may not be consistent when stating their preferences explicitly. This has the potential to make quantitative analysis of surveyed preferences difficult. The approach taken in this paper is to determine overall trends in preferences across a number of design alternatives, rather than assume that the findings for one alternative at one point in time are correct. Only the carafe selection problem was taken as a case study in this paper. The designers were provided additional features and specifications that might play a role in their preferences (Table 1).

Table 1. Sample of Design Information for Carafe Selection

| Name/ID | Glass carafe | Stainless-steel carafe | Plastic carafe |
| :--- | :--- | :--- | :--- |
| Photo |  |  |  |
| Description | Glass with warming plate | thermal-insulated stainless- <br> steel | thermal-insulated plastics <br> (inside glass) |
| Cost | $\$ 10.00$ | $\$ 20.00$ | $\$ 15.00$ |
| Warming plate cost | $\$ 5.00$ | 0 | 0 |
| Footprint size | Big | Small | Small |
| Fragility | Fragile | Strong | Fragile material inside |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |

Studies of team discussion suggest that team members enter discussion with only partial, independent knowledge of a topic. Group discussion can help in eliciting this partial knowledge so that higher quality decisions may be made (Gigone and Hastie 1997). In order to encourage discussion among the participants and to better simulate a real-world team experience, each participant were provided detailed information regarding only one of the three alternatives.
In this study, when preferential probabilities were extracted from the transcript using PPT, appraisal analysis rather than pure word occurrence was applied for better extracted results from the transcript (Honda et al. 2010). A base time interval of 10 minutes was used to calculate the preferential probabilities for two reasons: 10 minute intervals corresponded to the timing of surveys, and earlier work showed that 10 minute intervals resulted in a sufficient number of word occurrences for analysis. The first 3 intervals were used for analysis in this case study because designers finished selection of carafe in 3 intervals and the last one was focusing on continuous design of the filters.

### 4.2 Survey Ratings

The three designers are coded as $\mathrm{X}, \mathrm{Y}$ and Z . Table 2 details designers' relative survey ratings.
Table 2. Survey ratings from team members

| Interval | Designer | Carafe |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Designer X | Glass | Steel | Plastic |
| 1 | Designer Y | 0.5 | 0.3 | 0.2 |
|  | Designer Z | 0.6 | 0.4 | 0.1 |
|  | Designer X | 0.5 | 0.4 | 0 |
| 2 | Designer Y | 0.5 | 0.2 | 0.3 |
|  | Designer Z | 0.6 | 0.4 | 0.1 |
|  | Designer X | 0.5 | 0.3 | 0 |
| 3 | Designer Y | 0.6 | 0.3 | 0.1 |
|  | Designer Z | 1 | 0 | 0 |

### 4.3 Sampled Mappings

Using the method from 3.2, we first create a group of sample preference rating sets and run simulations to obtain corresponding preferential probabilities.
A step size is chosen that can accommodate the general case as well as edge cases. Too big a step size may cause significant errors when fitting the mapping from preferential probabilities to relative preference ratings; too small a step may impose strict requirements on the precision of the simulation and it may require a longer simulation run. In this case study, $1 / 12$ is chosen as the step size because it results in acceptable simulation time and tolerance error, and can cover edge cases when the ratings of 3 alternatives are equal or the ratings of 2 are equal.
Table 3 shows the sampled relative ratings for 3 alternatives and the corresponding preferential probabilities. Note it only includes the possible combinations of sampled ratings without permutation. "Alternative A" means the lowest rated alternative, not a certain alternative such as "Carafe Glass". The permutation of the samples can cover all cases. For example, the permutations for Row \#2 in Table 3 include $(0.000,0.083,0.917),(0.000,0.917,0.083)$, ( $0.083,0.000,0.917$ ), ( $0.083,0.917$, $0.000),(0.917,0.000,0.083)$, and ( $0.917,0.083,0.000$ ).

Table 3. Sampled Mapping from Relative Preference Ratings to Preferential Probabilities

| Sampled <br> Data | Relative Preference Ratings |  |  | Preferential Probabilities |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Alternative A | Alternative B | Alternative C | Alternative A | Alternative B | Alternative C |
| 1 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 1.000 |
| 2 | 0.000 | 0.083 | 0.917 | 0.000 | 0.003 | 0.997 |
| 3 | 0.000 | 0.167 | 0.833 | 0.000 | 0.050 | 0.950 |
| 4 | 0.000 | 0.250 | 0.750 | 0.000 | 0.140 | 0.860 |
| 5 | 0.000 | 0.333 | 0.667 | 0.000 | 0.258 | 0.742 |
| 6 | 0.000 | 0.417 | 0.583 | 0.000 | 0.375 | 0.625 |
| 7 | 0.000 | 0.500 | 0.500 | 0.000 | 0.000 | 1.000 |
| 8 | 0.083 | 0.083 | 0.833 | 0.004 | 0.004 | 0.991 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| 19 | 0.333 | 0.333 | 0.333 | 0.333 | 0.334 | 0.333 |

The Akima interpolation function in the statistics software package R was used to predict the relative ratings. When Akima interpolation is applied to any given vector of preferential probabilities ( $p_{l}, p_{2}, p_{3}$ ), $r_{1}$ can be obtained. And $r_{2}$ and $r_{3}$ can be retrieved by running interpolation on ( $p_{2}, p_{l}, p_{3}$ ) and ( $p_{3}, p_{l}, p_{2}$ ). As a sanity check, we make sure that ( $r_{1}, r_{2}, r_{3}$ ) sums up to 1 .

### 4.4 Assessment of Ratings Inverse-mapped from PPS

PPS estimates the preferential probabilities from individual survey ratings, so when they are mapped in the reverse direction by the proposed method to group relative preference ratings, the resulting ratings should lie within a reasonable range. The inverse-mapped ratings from PPS are shown in Table 4. All are between the maximum and the minimum of the individual survey ratings in Table 2.

### 4.6 Comparison of Ratings from Multiple Sources

Table 4 shows the relative preference ratings extracted from transcripts using Appraisal PPT, inversemapped from PPS, and average values based on individuals' preference ratings. The visual comparison is also illustrated in Figure 1. While the numeric values for these three are not exactly the same, the trends between the three methods are generally similar. All three methods show the glass carafe has the highest ratings in all three time intervals, the steel carafe has the second highest rating in all intervals, and the plastic carafe has the lowest rating throughout the discussion. This is also consistent with a qualitative reading of the transcript and consistent with the survey results.
Besides the general trend, we also notice fluctuations in preference ratings from Appraisal PPT during Session 2. A qualitative reading finds that the team was analyzing the features of steel carafe and plastic carafe in Session 2 after they had finished analyzing the glass carafe, and is consistent with these fluctuations.

Table 4. Group Relative Preference Ratings from Different Sources

| Time <br> Interval | Source of <br> Ratings | Preferential Probabilities for Carafe Selection |  | Mapped/Surveyed Relative Preference Ratings |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Glass | Steel | Plastic | Glass | Steel | Plastic |  |
| 1 | PPS | 0.654 | 0.322 | 0.023 | 0.562 | 0.353 | 0.084 |
|  | PPT | 0.615 | 0.270 | 0.114 | 0.511 | 0.299 | 0.192 |
|  | Avg Survey | - | - | - | 0.533 | 0.367 | 0.100 |
|  | PPS | 0.680 | 0.272 | 0.048 | 0.563 | 0.315 | 0.117 |
|  | PPT | 0.652 | 0.329 | 0.019 | 0.566 | 0.363 | 0.071 |
|  | Avg Survey | - | - | - | 0.533 | 0.333 | 0.133 |
|  | PPS | 0.884 | 0.101 | 0.015 | 0.734 | 0.203 | 0.063 |
|  | PPT | 0.860 | 0.086 | 0.054 | 0.686 | 0.182 | 0.133 |
|  | Avg Survey | - | - | - | 0.700 | 0.200 | 0.100 |



Figure 1: Relative Preference Ratings from Different Sources
The Pearson correlations and p-values between these 3 data sets (ratings from Appraisal PPT, ratings from PPS, and average survey ratings) are shown in Table 5 . We can see that correlation value between the ratings from PPS and the survey average ratings is very close to +1 , which indicates a statistically significant linear relationship. This makes intuitive sense because both are sourced from surveys, simply with different algorithms applied. This validates our assumption that the inversemapped ratings from PPS should be the same as the averaged group ratings when the team members contribute equally to discussion, or when there is no weighting information for individuals in the team discussion.

Table 5. Pearson Correlation Between Relative Ratings from Different Sources

|  | PPS Rating | PPT Rating | Survey Avg |
| :---: | :---: | :---: | :---: |
| PPS <br> Rating | - | 0.972 <br> $(\mathrm{p}$-value $=1.21 \mathrm{E}-5)$ | 0.999 <br> $(\mathrm{p}$-value $=3.39 \mathrm{E}-10)$ |
| PPT <br> Rating | - | - | 0.972 <br> $(\mathrm{p}$-value $=1.15 \mathrm{E}-5)$ |
| Survey <br> Avg | - | - | - |

The correlation between the ratings from Appraisal PPT and the ratings from surveys is statistically significant, which indicates that the ratings from the transcript could be a reliable source for implicit ratings, even when an explicit survey is not feasible or practical.

## 5 CONCLUSION

This paper examines the underlying theoretical mapping between preferential probabilities and relative preference ratings, and explores the feasibility of converting preferential probabilities into to relative preference ratings. The paper presents an approach for performing this conversion, and then illustrates the use of the approach in a case example. The method proposed in this research has the potential to greatly broaden the usability of preference information in a variety of applications. In particular, it
offers a possible way to link implicit preference information generated by design teams operating in the real world with formal design decision-making tools.
The approach in this work estimates the relative preference ratings from preferential probabilities by simulating under the principle of maximum entropy and interpolation for approximation. Rigorously speaking, the mapped preference ratings from preferential probabilities are estimated instead of calculated deterministically. These estimated ratings may be used in scenarios when other sources of preference ratings do not exist, are difficult to elicit, or are not reliable. The methodology explained in this paper can also be used as a check for preferences derived in other ways. When preferences from multiple sources are consistent, it offers confidence on the accuracy of the extracted preference ratings. By applying this approach to preference models which extract preferences in the form of preferential probabilities, it provides a clearer picture of how design selection can change over time. The case study in this paper focuses on preferences for design alternatives, but the proposed methodology can be applied to estimate preferences for the attributes of an alternative as well. A promising avenue for future research would be studying the effectiveness of employing the relative ratings extracted implicitly in establishing a decision making framework.
This work investigated a small group discussing a simple task. Many design problems in practice involve larger teams addressing more complex problems. Future work may consider team size, task complexity, the role of group dynamics, and team membership roles, such as authority, decisionmaking styles, and dominance of individuals. These would be useful directions for improving accuracy of forward- and inverse-mapping between relative preference ratings and preferential probabilities. The method in this research is based on simulation and statistical analysis, so uncertainty in the mapping process is also a valuable topic for future research. It could include subtopics such as evaluation and sensitivity analysis of the certainty/uncertainty of the mapped preference ratings.

## ACKNOWLEDGMENTS

The work described in this paper was supported in part by the National Science Foundation under Award CMMI-0900255. The opinions, findings, conclusions and recommendations expressed are those of the authors and do not necessarily reflect the views of the sponsors.

## REFERENCES

Akima, H. (1978). 'A Method of Bivariate Interpolation and Smooth Surface Fitting for Irregularly Distributed Data Points.' ACM Transactions on Mathematical Software 4(2): 148-159.
Arrow, K. J. (1970). Social Choice and Individual Values. New Haven, CT, US, Yale University Press. Arrow, K. J. and H. Raynaud (1986). Social Choice and Multicriterion Decision-Making. Cambridge, MA, US, The MIT Press.
Ben-Akiva, M. and S. R. Lerman (1985). Discrete Choice Analysis. Cambridge, Massachusetts, The MIT Press.
Box, G. E. P. and K. B. Wilson (1951). 'On the Experimental Attainment of Optimum Conditions.' Journal of the Royal Statistical Society (Series B) 13(1): 1-45.
Dong, A., M. C. Yang and T. Honda (2011). 'Time-Based Modeling of Linguistic Preference to Preferential Probability.' Proceedings of ASME 2011 International Design Engineering Technical Conferences \& Computers and Information in Engineering Conference.
Dym, C. L., W. H. Wood and M. J. Scott (2002). 'Rank ordering engineering designs: pairwise comparison charts and Borda counts.' Research in Engineering Design 13(4): 236-242.
Gigone, D. and R. Hastie (1997). 'The impact of information on small group choice.' Journal of personality and social psychology 72(1): 132-140.
Green, P. E. and V. Srinivasan (1990). 'Conjoint Analysis in Marketing: New Developments with Implications for Research and Practice.' Journal of Marketing 54(4): 3-19.
Hauser, J. R. and D. Clausing (1988). 'The House of Quality.' Harvard Business Review 66(3): 63-73.
Hazelrigg, G. A. (1998). 'A framework for decision-based engineering design.' Journal of mechanical design 120(4): 653-658.
Hensher, D. A. and L. W. Johnson (1981). Applied Discrete Choice Modeling. New York, Halsted Press.
Hey, J. D. (1998). Do Rational People Make Mistakes? Game Theory, Experience, Rationality. W. Leinfellner and E. Kohler. Netherlands, Kluwer Academic Publishers: 55-66.

Honda, T., M. C. Yang, A. Dong and H. Ji (2010). A Comparison of Formal Methods for Evaluating the Language of Preference in Engineering Design. International Design Engineering Technical Conferences \& Information in Engineering Conference, Montreal, Canada.
Howard, R. A. and J. E. Matheson (1984). The Principles and Applications of Decision Analysis. Menlo Park, CA, Strategic Decision Group.
Jabeur, K., J.-M. Martel and S. B. Khelifa (2004). 'A Distance-Based Collective Preorder Integrating the Relative Importance of the Group's Members.' Group Decision and Negotiation 13(4): 327-349.
Jaynes, E. T. (1957). 'Information Theory and Statistical Mechanics.' Physical Review 106(4): 620-630. Jaynes, E. T. (1968). 'Prior Probabilities.' IEEE Transactions On Systems Science and Cybernetics sec4(3): 227-241.
Ji, H., T. Honda and M. C. Yang (2011). 'Granularity Enhancement of Extracted Preferential Probabilities from Design Team Discussion.' Proceedings of ASME 2011 International Design Engineering Technical Conferences \& Computers and Information in Engineering Conference.
Ji, H., M. C. Yang and T. Honda (2007). 'A Probabilistic Approach for Extracting Design Preferences from Design Team Discussion.' Proceedings of ASME 2007 International Design Engineering Technical Conferences \& Computers and Information in Engineering Conference.
Ji, H., M. C. Yang and T. Honda (2012). 'An Approach to the Extraction of Preference-related Information from Design Team Language.' Research in Engineering Design 23(2): 85-103.
Keeney, R. L. (1976). 'A Group Preference: Axiomatization with Cardinal Utility.' Management Science 23(2): 140-145.
Keeney, R. L. and H. Raiffa (1976). Decisions with Multiple Objectives: Preferences and Value Tradeoffs. New York, Wiley.
Krantz, D. H., R. D. Luce, P. Suppes and A. Tversky (1971). Foundations of Measurement Volume 1. New York, NY, US, Academic Preess.
Kulok, M. and K. Lewis (2005). 'Preference Consistency in Multiattribute Decision Making.' ASME Conference Proceedings 2005(4742Xa): 291-300.
Otto, K. N. and E. K. Antonsson (1991). 'Trade-Off Strategies in Engineering Design.' Research in Engineering Design 3(2): 87-104.
Otto, K. N. and E. K. Antonsson (1993). The Method of Imprecision Compared to Utility Theory for Design Selection Problems. ASME 1993 Design Theory and Methodology Conference.
Press, W. H., S. A. Teukolsky, W. T. Vetterling and B. P. Flannery (2007). Numerical Recipes: The Art of Scientific Computing. New York, Cambridge University Press.
Pugh, S. (1991). Total design: integrated methods for successful product engineering. Wokingham, England, Addison-Wesley.
Ross, S. M. (2006). Simulation. Burlington, MA, Academic Press.
Saaty, T. L. (2000). Fundamentals of Decision Making and Priority Theory With the Analytic Hierarchy Process. Pittsburgh, RWS Publications.
Scott, M. J. and E. K. Antonsson (1998). 'Aggregation functions for engineering design trade-offs.' Fuzzy Sets and Systems 99(3): 253-264.
See, T.-K. and K. Lewis (2006). 'A Formal Approach to Handling Conflicts in Multiattribute Group Decision Making.' Journal of Mechanical Design 128(4): 678-688.
Thurston, D. (1991). 'A Formal Method for Subjective Design Evaluation with Multiple Attributes.' Research in Engineering Design 3(2): 105-122.
Tribus, M. (1969). Rational Descriptions, Decisions, and Designs. New York, Pergamon Press.
von Neumann, J. and O. Morgenstern (1947). Theory of Games and Economic Behaviour. Princeton, Princeton University Press.
Vroom, V. and A. Jago (1988). The New Leadership: Managing Participation in Organizations. Upper Saddle River, NJ, Prentice-Hall.
Wang, J. (1997). 'A Fuzzy Outranking Method for Conceptual Design Evaluation.' International Journal of Production Research 35(4): 995-1010.
Wood, K. L. and E. K. Antonsson (1989). 'Computations with Imprecise Parameters in Engineering Design: Background and Theory.' ASME Journal of Mechanisms, Transmissions, and Automation in Design 111(4): 616-625.

