# ROBUST DESIGN PROPOSAL BY THE USE OF STRUCTURAL TOPOLOGY OPTIMIZATION CONSIDERING UNCERTAINTIES OF INPUT PARAMETERS AND BOUNDARY CONDITIONS

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## ABSTRACT

Specifying a product's optimal mechanical and functional design primary depends on the requirements defined by the future operating conditions. In today's product design process the usage of topology optimization is a widely applied computer aided method to define an optimal design considering these requirements. However, in an early design stage, input parameters for this structural optimization process (e.g., operation conditions) are usually uncertain. Consequently, the resulting design is not optimal.

This paper focuses on a detailed investigation of uncertainties in structural topology optimization. Hence, the different kinds of uncertainties and their effects on the resulting designs are considered in detail. Furthermore, a methodology is presented, which enables the consideration of uncertainties using e.g., a statistical investigation of the topology optimization results and a sensitivity analysis-based result visualization. Therefore, a cam drive rocker arm of a combustion engine is analyzed in a case study. Finally, several recommendations are derived, supporting the product developer to define a robust initial design, causing less computational and time expense.

Keywords: uncertainty in topology optimization, early functional validation, robust design, optimisation, design engineering

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# **1** INTRODUCTION

Today, the short product lifecycle is a main challenge to the product developer. This requires a high level of product maturity – especially in the virtual design stage. Additionally, the growing focus on lightweight design, forces the product developer to define a product design, which ensures both the given functional requirements as well as it presents an optimal ratio of stiffness to weight. To fulfill these requirements and to reduce/avoid design iterations, the early application of computer aided design and simulation tools for successful and economical product development is a common practice today. Therefore, the product developer uses methods and tools like structural topology optimization for gaining an optimal initial design and thus, ensures the product's functionality and the lightweight design aspects in an early design stage (Müller et. al., 1999).

A common topology optimization task, resulting in an optimal material distribution within a given design space, is to maximize stiffness (minimize weighted compliance) as objective function and minimize mass (meet a volume constraint) for a given set of loads and boundary conditions.

In any case, an analysis model is the basis for the structural optimization, which can be both based on analytical and numerical methods like the finite element method. However, a topology optimization enables the product developer to compute a discrete geometry of the considered component that meets the given mechanical requirements (Bendsøe and Sigmund, 2005). In this context, the topology optimization is often seen as the "holy grail" for generating an optimal structural design. But considering uncertainties of input parameters and boundary conditions, this structural optimum could only be a compromise to meet the given, usually diverging requirements.

The product's functional requirements are defined in the first stage of the product development process, while its specified concept is detailed subsequently (second stage, see also Pahl and Beitz, 2007). This forces the product developer to choose the final input parameters for the analysis model during the early stages of the product development process. These specific values and value ranges of miscellaneous influence parameters, such as loads, material properties, geometrical dimensions and deformations fully depend on future operating conditions of the resulting product.

Hence, the product developer has to face the challenge, to clearly define the input parameters for the structural optimization process considering operation uncertainties. Nevertheless, these uncertainties and their impact on the optimization's results are difficult to evaluate. Consequently, the following research questions arise:

- Which uncertain parameters should be taken into account to determine a robust initial design (considering lightweight design aspects) by means of the topology optimization?
- How much contributes the variation of a specific input-parameter to the variation of the resulting design's functional key properties?
- How can several slightly different design proposals (resulting from multiple optimization runs with different parameter combinations) be combined to a final robust design proposal that meets the given operating requirements?

This paper focuses on answering these questions, by presenting a design methodology, which uses the topology optimization to determine a robust mechanical design of a single part under e.g. varying operation conditions and simulation parameters. Subsequently, the benefit of the approach is a reduction of time- and money-consuming iterations by starting with a robust initial design in the early design process (Walter et. al., 2011). Therefore, it is essential to understand the influences of uncertainties on a structural optimization in detail. As a result, the product developer is able to identify and to quantify these factors and their impact as well as to ensure that the optimized structure's design meets all given operating conditions. Furthermore, it is shown that each optimization model has got several similar input parameters heavily influencing the design result, but could be considered constant for different simulations. Consequently, several recommendations for topology optimization tasks are derived for the product developer.

# 2 STATE OF THE ART

The common proceeding of the product developer performing a topology optimization of a part is to use a deterministic approach, assuming that all input parameters can be determined exactly. So even known varying parameters are chosen as constant values by best knowledge. But this approach faces some major problems:

- The optimization result is determined without consideration of various uncertainties.
- The influence of varying loads, boundary conditions, material properties or geometric variations on the resulting structure's performance are not taken into account.
- The optimized structure of the considered part is possibly not consequently a robust design, which means that the design doesn't meet the design requirements and is sensitive to e.g. manufacturing errors or other sources of uncertainties.

These problems lead to a non-optimal design proposal considering the case that the design should be robust for complete operating range of the product. To avoid these problems several approaches for directly integrating a stochastic or reliability based formulation into the topology optimization algorithm have been developed. For instance, Guo et al. (2013) presented a bi-level formulation for robust topology optimization considering the uncertainty of boundary variations. Chen et al. (2010) developed a level set based robust shape and topology optimization with consideration of random field uncertainty in loading and material properties. To address the topology optimization considering manufacturing errors of micro-structures, Sigmund (2009) presented a morphology-based filter method. The examples presented in these publications demonstrate that promising robust designs can be obtained by these approaches. But topology optimization considering uncertainties is still an open research area, which needs further investigation. The mentioned approaches have not yet been implemented in commercial structural optimization products and thus are not available to the product developer. Also these integrated methods would be a "black box" that does not show which uncertain parameters should be taken into account and how much influence the deviation of a specific parameter has on the result. So uncertainties and their importance in relation to the optimization results are still difficult to evaluate for the product developer.

# 3 METHODOLOGY

This paper provides a methodology to the product developer for obtaining a structural robust design of a part of a mechanism based upon topology optimization with uncertain input parameters and boundary conditions. To illustrate the design approach an optimization study of a valve train rocker arm inside a four-stroke, single-cylinder combustion engine is done in section 4.

The methodology to determine a robust design proposal of the mechanism's part under uncertain conditions by means of structural topology optimization follows eight steps (Figure 1):



Figure 1. Methodology to determine a robust design proposal by structural topology optimization under uncertain conditions

In the first step, based on the product concept or an existing component, the design space for the topology optimization problem needs to be defined (section 4.1). In order to identify the scattering parameters, the manufacturing process should be known. Moreover the shape of the design space has to be specified taking into account manufacturing constraints. Eventually, important input parameters and their nominal values need to be identified, classified and converted to analysis-model-parameters in the second step (first part of section 4.2). In addition to the nominal values all uncertainties, for example manufacturing-caused deviations, operating-depending loads and constraints, material properties and analysis parameters need to be evaluated to determine the limits and distributions of the

varying model parameters. In order to investigate the effects of varying input parameters on the optimization's design result the provided methodology uses the design of experiments (DOE). So the next essential step is to define multiple optimization tasks for the specified uncertainty ranges of the model parameters by using a parameter sampling (second part of section 4.2).

Based on the input parameter definition of the samples and the component's design space from the first step, a parametric finite element analysis model can be implemented in step four (section 4.3, Pre-Processing). This parametric model is used to define a single finite element model and setup a topology optimization task for each design point of the sampling during the fifth step (section 4.3, Processing). To generate a response for the sensitivity analysis, each result structure of the optimization has to be setup as a finite element analysis model and to be simulated with the nominal load case of the input parameters (section 4.3, Post-Processing).

In step seven the two analysis approaches (section 4.4 and 4.5), the sensitivity analysis based upon meta-modeling and the visual interpretation yield both distinctive input parameters and selective optimization shapes. The filtered structures are finally recombined and reconstructed by using CAD to a functional robust design proposal (section 4.6). A last optional step is the validation of the reconstructed robust design proposal by simulating multiple operating points, for the extreme points of the varying input parameters, which is also presented in section 4.6 for the use case.

Based on the insights of the study several recommendations for topology optimization considering uncertainties in the design process are presented in section 5.

# 4 CASE STUDY

# 4.1 Demonstrator: Valve train rocker arm and design space

To demonstrate the methodology in practical use, an optimization study is performed. This study focuses on the rocker arm of a valve train, shown in Figure 2, in a four-stroke single-cylinder diesel engine. Its bore diameter is 85 mm and the stroke is 90 mm. Two overhead camshafts operate one exhaust and one intake valve via rocker arms. The material of the forged rocker arm is wrought aluminum alloy, e.g. EN-AW-2014 (AlCu4SiMg) with a minimum specified fatigue strength of  $100 \text{ N/mm}^2$  for  $10^7$  alternation loadings (taken from manufacturer's material specifications, e.g. Alu Menziken Extrusion AG) – which equates the required operating time of the engine.

The main challenge is to maximize stiffness of the design – maximum deformation should be smaller than 150  $\mu$ m to avoid significant effects on the valve timings (deformation of current design under nominal load is about 330  $\mu$ m) – and simultaneously reduce mass (lightweight design). Furthermore the maximum tension of the robust design proposal should not exceed the minimum specified fatigue strength for the complete operation range.

As mentioned in section 3, a design space is needed for the topology optimization, which is also shown in Figure 2 for the rocker arm (orange component).



Figure 2. Valve train demonstrator and design space of the rocker arm (exhaust side)

Based on the objective (maximize stiffness), the design space should be maximized. But especially for dynamic systems, collisions with surrounding components should be avoided during the entire motion. With regard to complex dynamic systems, tools like multi-body-dynamic simulations could be used to determine the available free space. To define the design space for the rocker arm, 2D sketches of the extreme positions (valve: completely opened and closed) are considered (Figure 2). The depth of the design space corresponds to the depth of the original rocker arm.

#### 4.2 Definition and sampling of the valve train's varying parameters

During one motion sequence of the combustion engine, the valve train's camshaft rotates with a total angle of  $\varphi_{CS} = 360^{\circ}$ . Beside the varying operation parameters, all necessary input parameters and the expectable deviations (e.g. manufacturing-caused deviations) have to be listed. These varying input parameters can be categorized in two types, "assignable and unassignable parameters", which will be further detailed in the upcoming sub-section. Figure 3 details the case study's considered varying parameters.



Figure 3. Varying parameters of the rocker arm

#### Assignable, unassignable and replacement parameters

Assignable parameters can be defined as "parameters of a FE-model characterized by constant values for both the dynamic and static state of a system". In contrast, unassignable parameters are defined as "parameters of a FE-model that change their values for the different operating points of the system".

Parameter	Description	<b>Reason for variation</b>	Value range
$L_{SH}$	Distance between bush bearing bore and tappet bore.	A deviation of this parameter may occur due to inaccurate manufacturing conditions	73.9 mm ±2 %
g <sub>e</sub>	Element size of the finite element model.	Mesh refinement increases accuracy of the analysis model and perhaps of the result structure, but also increases computing time	0.5 mm – 5 mm
E	Young's modulus	Deviations of these parameters may result from structural changes	65 GPa – 75 GPa
ν	Poisson's ratio	of the material during the production process	0.32 - 0.35
$\mu^*$	Coefficient of friction between cam and rocker arm	Friction occurs between the relative movement of the cam and the rocker arm. Lubrication, surface roughness and pollution are important influences	0.01 – 0.3
$\varphi_{CS}^{*}$	Angular position of the cam	The cyclic rotation of the cam defines the position of the rocker arm and the valve. The result is a function of time-varying load conditions	-90° – 270° nominal 91.1°
$\gamma_x^*$	Angle of the force vector to the x-axis	A manufacturing-caused angle error of the bush bearing bore can result in an inclination of the valve spring force	$0^{\circ} \pm 2^{\circ}$

Table 1. Assignable and unassignable parameters (marked with \*)

Both assignable and unassignable parameters follow uniform distributions, except the bore distance  $L_{SH}$ , the angular position of the cam  $\varphi_{CS}$ , which are distributed normally and furthermore the angle of the force vector to the z-axis  $\gamma_z$  which results from the angular position of the cam.

The topology optimization only allows static analysis (Müller et. al., 1999). Hence, the complete definition of the required finite-element model (FE-model) requires the replacement of unassignable parameters by so-called "replacement parameters". These replacement parameters can be applied to the FE-model. Therefore, the mathematical and physical relationships as well as the kinematic behavior of the valve train need to be considered. Table 2 details the replacement parameters of the rocker arm.

The nominal values of the parameters result from the maximum deflection of the rocker arm. At this camshaft position ( $\varphi_{CS} = 91.1^{\circ}$ ) the load conditions of the rocker arm reach their maximum values.

This load case will be used for the validation analysis to obtain the responses (volume, deformations of point C and maximum stress), which are investigated during the sensitivity analysis.

Parameter	Description	Reason for variation	Value range
$(F_x, F_y, F_z)$	Single values of the force vector to the main directions.	The force vector in global directions $(F_x, F_y, F_z)$ is needed for the FE-model and depends on $(\varphi_{NS}, \gamma_x, \gamma_z)$ .	$F_{x:}$ -123.6 N - 14.7 N $F_{y:}$ 540.6 N - 1139.7 N $F_{z:}$ -34.2 N - 38.6 N
$x(\varphi_{CS})$	Position of the line contact.	Due to the given geometry, the position of the line contact between cam and rocker arm can be determined in dependence of the angular position of the cam.	37.20 mm – 56.63 mm
$R_b(\mu, \varphi_{CS})$	Frictional force at the line contact.	The absolute value of $R_b(\mu, \varphi_{CS})$ results from the vertical reaction force $B_y$ at the line contact and the friction coefficient $\mu$ .	-556.30 N – -18.05 N

Table 2. Replacement parameters

## Parameter Sampling

In order to generate a statistical reliable data-set, the topology optimization of the rocker-arm has to be performed several times. Consequently, a destined number of "virtual" value trains are generated – the so-called samples – using Latin-Hypercube-Sampling (LHS) (McKay et al., 1979). These samples just differ in the values of the assignable and unassignable input-parameters. The number of required samples depends on two diverging requirements: Many samples should be generated to ensure good and reliable results. However, a large number of samples results in increasing computational expense. Hence, 50 samples of the non-ideal value train are generated. This number causes reasonable numerical expense, but still allows the determination of statistical reliable meta-models in section 4.4.

## 4.3 Statistical topology optimization of the rocker arm

According to the methodology (Figure 1), the topology optimization of the rocker arm has to be performed for each of the previously generated 50 virtual non-ideal valve trains. This procedure can – similar to a classic FE-Analysis – be divided into three main steps: Pre-Processing, Processing and the Post-Processing. Consequently, the first step includes definition and generation of an appropriate FE-Model, which "virtually" reproduces the given situation of the rocker arm. Moreover, in order to establish a statistical topology optimization, this model should be parameterized to be easily adapted to each sample's input-parameters.

## **Pre-Processing**

This step includes the definition of a FE-Analysis model, including loads, boundary conditions, material properties and non-design areas of the design space (using ANSYS Workbench 14). The information, needed to setup up the parametric model, is available (Table 1 and 3). Figure 4 details the analysis model's parameters and the chosen design space.



Figure 4. Analysis model definition of the rocker arm

Two simplifications are applied: The constraint in the bush bearing bore (A) is considered frictionless and the contact surface of the cam is assumed to be a frictionless line contact (B), where the friction force  $F_b$  of the cam is applied. The main load vector (with its components  $F_x$ ,  $F_y$ ,  $F_z$ ), resulting from the valve spring force, lasts on the tappet bore (C). Also non-design areas (grey colored areas in Figure 4) are defined to ensure functional required areas, like the bush bearing. Due to the possibility of an automatic meshing, an area-dependent tetrahedral mesh-method (quadratic approach) is chosen.

#### Processing

Each FE-model requires a semi-automatic topology optimization setup (using TOSCA.Structure 7.1). The objective function of the optimization task is to "minimize weighted compliance of the structure" while not exceeding a given lower volume constraint of 14,000 mm<sup>3</sup>. Furthermore, a demold constraint to the design space (along the global z-axis) is defined to ensure the optimized design being manufactured as a forged part. Subsequently, the 50 optimization jobs (one job for each sample) are processed by a batch queue and also the result structures – the smoothed iso-surfaces of the topology optimization density results - are exported as triangulated surface meshes.

#### Post-Processing

The smoothed result structures of the optimization are redefined as FE-models in order to deliver the design responses for the result representation and interpretation (section 4.4). The design responses, determined by the validation analysis, are derived from the objectives of the topology optimization:

- Maximum deformation (along z- and y-axis) of the tapper bore in point C (see Figure 4)
- Maximum mean stress (von Mises) within the optimized structure
- The total volume of each resulting structures

## 4.4 Result representation and interpretation using meta-models

The statistical topology optimization results in 50 different design proposals of the rocker arm. However, finally only a single geometry of the rocker arm can be manufactured and thus, is required. Consequently, these 50 different structures must be combined; or in other words, the most essential geometrical characteristics of each structure must be identified and merged into a final geometry.

Therefore, two different methods are used: On the one hand, a visual investigation and interpretation of the resulting 50 meshes is detailed in section 4.5. On the other hand, a variance-based global sensitivity analysis can quantify the contribution of each input-parameter's variation on the rocker arm's varying responses/characteristics (e.g., volume, appearing deformation).

The global sensitivity analysis allows the determination of the main effect as well as the total effect  $S_{Ti}$ of a varying input-parameter of the topology optimization. The main effect quantifies the effect and thus, the sensitivity of a parameter towards an output parameter, while the total effect considers these parameter's interactions with additional parameters, too (Saltelli et al., 2000). Higher total effects correspond with a significant influence of the considered parameter. However, since a sensitivity analysis usually requires far more samples than the 50, which are available, an appropriate meta-model is generated. According to Kleinjen (2009), a meta-model is "an approximation of the multiinput/multi-output relation given by the simulation model". Consequently, any number of samples can be generated by means of the meta-model and thus, the sensitivity analysis can be performed. In this case, Artificial Neural Networks are used. An Artificial Neural Network (ANN) consists of several neurons and connections among them. The neurons are placed on one or more layers - the so-called hidden layers – and their importance is evaluated by synaptic weights. For a more detailed description on the ANNs' use for variance-based sensitivity analyses, see Walter et al. (2012). Certainly, since meta-models are just approximations and thus, cause prediction errors, their prediction quality must be evaluated. Therefore, the so-called coefficient of prognosis (COP) is determined (Most and Will, 2008). Table 3 details each response's ANN-settings as well as the corresponding COPs.

Output parameter	Neurons on first layer	Neurons on second layer	СОР
volume	9	(no second layer)	87.78 %
deformation Defy	18	12	78.97 %
deformation Def <sub>Z</sub>	18	12	91.24 %
maximum stress	18	9	67.70 %

Table 3. Settings and prediction qualities of Artificial Neural Networks

The presented total effects (sensitivities), based on Sobol's approach with 100,000 samples using Eikos (Toolbox of Matlab), are shown in Figure 5. As detailed in 4.3, the output-parameters are the final volume of the mesh, the geometry's deformations of point C along the Y- and Z-axes ( $Def_Y$  and  $Def_Z$ ) as well as the appearing maximum stress.

It can be seen that the cam angle  $\varphi_{CS}^*$  (high total effect of up to 0.633) and  $\gamma_x^*$  (0.258 and 0.547) mainly influence the deformation of the resulting structure, whereas particularly the element size  $g_e$ 

influences the resulting volume ( $S_{T,element_size} = 0.777$ ). Only the friction coefficient  $\mu^*$  has a slightly higher influence on the appearing maximum stress than all other input parameters ( $S_{T,\mu} = 0.406$ ).



Figure 5. Total effects  $S_{Ti}$  of the seven input-parameters on the four output-parameters

#### 4.5 Visual result interpretation

Only qualitative statements about the selection of the most essential structures are possible, because of the not always sufficiently high COPs. So a manual visual investigation and interpretation is used to supplement the previous insights. Basic similarities and characteristics between the structures can be determined. Because of this, it is possible to classify and summarize the result structures in six categories. Table 4 exemplarily shows representative result meshes for each category.



Table 4. Topology optimization result categories

Several influences on the decision, whether the structure is a "good" or a "poor" result should be considered. Factors, such as shape, resolution of details, stiffness, manufacturing-oriented design or maximum stress, should be taken into account. The category's numbers are arranged in a quality-order – starting with "poor designs" (Categories 1 and 2) and ending with the "good design"-categories 5 and 6. As already seen in section 4.4, especially the parameter "element size" has a significant influence on the volume and thus, the lightweight design-aspects. Moreover, results with a "high resolution", like category 5 and 6, are mainly characterized by a high mesh density. For this reason, the "poor" categories, only show a low resolution and don't meet the volume restriction.

Therefore, only the structures of category 5 and 6, with an element size smaller than 1.5 mm approximately meet the volume restriction and show a high level of detail. Furthermore, the structures of categories 1-3 lead to poor mechanical properties. For example comparatively low stiffness or

high stress peaks due to adverse connections within the shape. Basically, several structures of category 4 show positive mechanical properties, but only provide a moderate resolution of details. Structures of category 5 and 6 have the most robust mechanical properties. Thus only these two result categories are considered as ideal for a robust design reconstruction (upcoming section).

## 4.6 Result combination and design reconstruction

Based on the knowledge of the result interpretation, the most suitable design points of the sampling can be determined for design reconstruction. According to this, only samples with an element size smaller than 1.5 mm, which are not dedicated to category 1 to 4 and which are not disqualified by structurally illogical design characteristics, will be considered.

To obtain robust design proposals, the remaining ten meshes of the categories 5 and 6 are merged manually into a functional and manufacturing-oriented design, by the use of a CAD-System. The reconstructed robust design proposals of each category, as well as the corresponding maximum stresses (under nominal load conditions with a cam angle of  $91.1^{\circ}$ ), are shown in Figure 6.



Figure 6. Robust design proposals of the rocker arm, left side Cat. 5, right side Cat. 6

It turns out that the occurring stress of the robust design proposal of category 6 is about 25 % lower. Consequently, only the merged geometry of category 6 is considered for further investigation. To confirm the robustness of the geometry further simulations are performed at different angular positions of the camshaft and worst-case operating conditions ( $\mu = 0.3$  and asymmetric load relative to the x-axis). Exemplarily in Figure 7 the stress results for two extreme cam angles are shown.



Figure 7. Robustness validation of the design proposals

The maximum stress does not exceed the minimum specified fatigue strength of 100 N/mm<sup>2</sup>. Moreover, the deformation of point C does not exceed the specified limit of 150  $\mu$ m and thus, the provided design proposal can be seen as robust. Furthermore there is a clear improvement over the original design: mean stress was reduced by the factor 4, the deformation in y-direction is only about one third of the original geometry, except the deformation in z direction, which is about 25 % higher. Also a mass reduction of 6 % was achieved.

# 5 RECOMMENDATIONS FOR TOPOLOGY OTPIMIZATION CONSIDERING UNCERTAINTIES OF INPUT PARAMETERS

Based upon the provided methodology and the insights during the case study, several recommendations for the product developer can be derived:

- The definition of the boundary conditions and load case of a static FE model for a dynamic system (with varying boundary conditions and load cases) should represent the extreme positions with the maximum load case (worst-case-analysis).
- Local changes of the boundary conditions for different load cases must be avoided and should always represent the nominal position (maximum load case) of the dynamic system.
- Rough meshed FE-models should be optimized, since these allow a checkup if the selected load

and boundary conditions are appropriate.

- If the model is reasonable, a high mesh density is required to guarantee a sufficient detail resolution of the optimized structure.
- Prevent adverse forms by using the "minimum member size control" as constraint for the topology optimization, especially when using a very detailed mesh. This ensures, that the result structures are suitable for manufacturing and do not contain thin links causing high stress peaks.

In order to validate the recommendations another topology optimization of the use case with all essential aspects of the topology optimization is performed. The analysis model is simulated by using the nominal input parameters (cam angle  $91.1^{\circ}$ ) except the following:

Element size < 0.7 mm (1.2 million elements); minimum membersize control = 3 mm; volume constraint = 12,000 mm3. Figure 8 shows the result structure of the optimization provided by the recommendations and the robust design proposal, which was detailed in Figure 7. In conclusion it can be said that the optimization result considering the recommendations, is already very close to the robust design proposal. So the next step would be to use this work's result structure for the provided methodology for further improvement of the geometry without varying geometric dimensions or boundary conditions to improve the meta-models' prediction qualities.



Figure 8. Optimization result provided by recommendations and robust design proposal

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