13 MODELING PROPERTIES FOR THE DESIGN OF BRANCHED SHEET METAL PRODUCTS

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This paper presents an approach to designing multi-chambered profile structures made by a new manufacturing process called linear flow splitting using an algorithm based approach. The basis for this procedure is a systematic view of product properties and the separation of its internal and external properties. The link between product properties must be derived by design knowledge such as physical models or guidelines. This specific knowledge enables one to decide which internal properties, as optimization parameters, need to be adjusted in order to meet desired external properties. This procedure leads to mathematical problems that are difficult to solve, presenting a challenge for research. This approach is demonstrated with an example of a development assignment seeking a topology of a profile structure consisting of several chambers.

Keywords: Algorithm-based design, Product properties, Optimization, Sheet metal.

1. INTRODUCTION

The product development process is shaped by the know-how and the intuition of the designer. Consistent computer aid is established in some elementary work procedures, but it is typically missing in the early phase of the product development process. The main focus of the new algorithm-based approach is to elaborate the customer- and market requirements for profile structures in such way, that mathematical optimization processes can follow. The difficulty is developing a product which exactly fulfills these requirements by the product's outward properties, called external. The vague customer statements must be prepared in such a way that they can be used as parameters in mathematical-algorithmic optimization procedures for the following topological and geometrical design of sheet metal profiles. New procedures for an algorithm-based product development are prepared in close co-operation between mathematicians and designers. The core element of the Collaborative Research Center 666 (CRC 666) (German: Sonderforschungsbereich 666, SFB 666) is made up of new forming processes called linear flow splitting and linear split bending. Linear flow splitting allows branched profiles to be formed using sheet metal in integral style without joining, laminating material or heating the semi-finished part. Since this forming technique is at an early stage of development, a way to integrate it into the product design process must be developed. Therefore, the goal of the CRC 666, which was founded in 2005, is to exemplify how branched sheet metal products can be designed and how a manufacturing process can be integrated into the design process.

1.1. Linear Flow Splitting

Integral sheet metal structures made by linear flow splitting show great potential for producing innovative and lightweight products. In nature, branched structures are used for numerous purposes, e.g. to provide stability in a leaf. So far, branched sheet metal structures were mainly produced in differential style, i.e. by gluing, welding or similar procedures or by material sheeting. These procedures have several disadvantages: they are heavier, have lower thermal conductivity, a higher disposition to corrosion and are more likely to break due to instability at the connecting piece or the double layer. At the present time, the stabilizing branches in the wings of airplanes are milled out of huge blocks of material; stringer constructions in the shipbuilding industry are welded onto the material, which is time consuming and leads to a change in the microstructure of the welding joint. Manufacturing those parts with integral style branches by using linear flow splitting[1] could eliminate these drawbacks. It is a new massive forming process for the production of bifurcated profiles in integral style. The semi-finished part is a sheet metal plane, which is transformed at ambient temperature by a specific tooling system which consists of obtuse angled splitting rolls and supporting rolls (Figure 1, (1)). The fixed tool system forms the translatory moved work piece in discreet steps up to a profile with the final geometry (Figure 1, (2)). The further processing of the split sheet metal by roll forming and bending procedures presents the opportunity to produce multi-chambered profiles with new cross-sections from sheet metal (Figure 1, (3) and (4)). Using renewed linear flow splitting of the end of the flange and forming of the profiles, numerous new possibilities for chambered profiles optimizing lightweight design arise.[2]

2. PRODUCT DESIGN AS AN OPTIMIZATION OF INTERNAL PROPERTIES

Product design is a process of variation and selection over different phases by adding product properties with every step and becoming more and more concrete.[3] The general design process starts with the definition of user needs and desires, also called user requirements that are partly specified in the non-technical vocabulary of the users. The work result should be a requirement list. It comprises requirements which a product should fulfill. Designers have the task of meeting these requirements using the product's outward properties. The definition of properties, as a set of an attribute and a related value (with or without dimension) related to a product is fundamental for design science and design methodology. Products differ if at least one property differs which means that either a new attribute can be seen (a heat exchanger without cooling rips) or another value for the same attribute creates a new product (a heat exchanger with the length of 1.6 meter).

2.1. Internal and External Properties

The external properties of a product are its outwardly perceivable properties and represent properties of the product the customer can observe, recognize and judge (e.g. stiffness, durability). An external property of a product has to be realized by the exact determination of internal properties and never exists in itself.[4] An external property it is always the result of internal properties (e.g. geometry, material). This means designers must choose internal properties in such a way that the required external properties are met as closely as possible. Everything which is changed in a technical drawing, e.g.

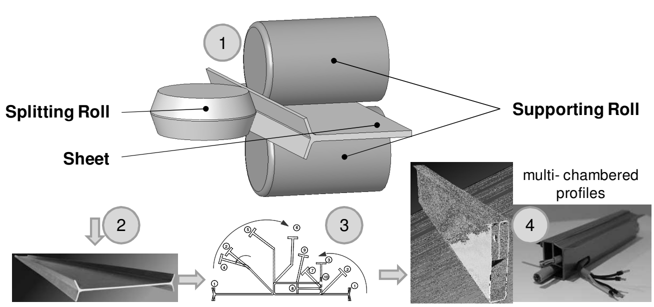

Figure 1. Manufacturing multi-chambered profiles.

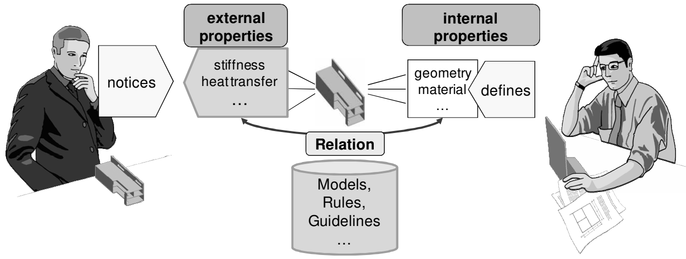

Figure 2. Optimization of Internal Properties to achieve External Properties.

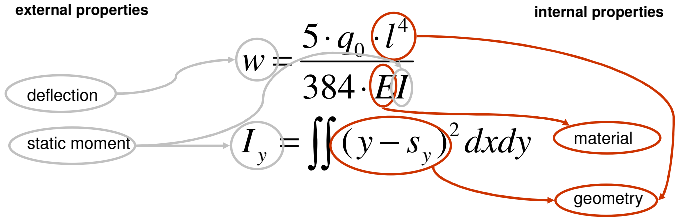

Figure 3. Formula of Beam Bending classified into External and Internal Properties.

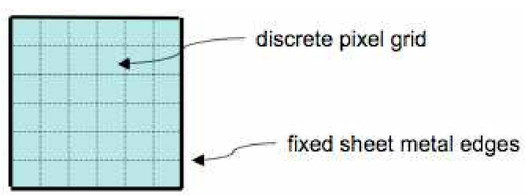

Figure 4. The Discretized Design envelope (grey).

geometric data, certain features or the material — all internal properties — has a direct effect on the external properties perceived by a customer.

The class of external properties comprises all product properties the product possesses such as fluid dynamic properties and mechanical properties and if we consider the product's environment and the process it is involved in (e.g. heat exchange) as well as behavior of the product. Internal properties represent "setscrews" that a designer can adjust. They can be used as optimization parameters (e.g. geometry and material parameters) to achieve a required external property (e.g. deflection of a profile).

2.1.1. Design Knowledge to connect Internal and External Properties

The knowledge of the correct correlation between internal and external properties is of crucial importance for the designer to understand the internal mechanisms of a product. This design knowledge enables the designer to optimally adapt the product and its properties to the desired requirements. The knowledge of relations between properties are often expressed in scientific literature by physical models (e.g. friction model, beam bending model), rules, design principles, or design guidelines (e.g. design guideline with examples for bend parts or welded components).[5] These sources of information are categorized and analyzed in this paper, with special regard to product properties and their relations to each other. For example, the stiffness noticed by the customer stands in relation to the internal properties selected by the designer such as material or topology data. The challenge is for a designer to create and optimize the desired external properties using the products internal properties. Therefore the designer needs certain design knowledge. To achieve an algorithm-based design this knowledge has to be implemented into mathematical optimization routines. North[6] defines knowledge as the entirety of cognitions, skills and experience, that someone uses to solve a problem. Liese,[7] among others, undertakes a structuring of sources in which knowledge is saved and stored. He differentiates between elementary and aggregated sources. Elementary sources cannot be broken down any further. Some examples are formulas, texts, lists or geometry elements. Aggregated sources result from the synthesis of the elementary sources e.g. rules, guidelines or DIN standards. If we describe the relationship between properties with a formula we assign one property to another. For the argument x of a formula one can select an internal property which directly possesses an external property as its function value. A formula without a correlation to technology or nature usually has little meaning for the designer. It does not set up a relationship between internal and external product properties.

2.1.2. Physical Model of Beam Bending

In the early phases of design formal, physical connections play a particular role. The link between internal and external properties is often given by models like the beam deflection model, which consists of general mechanic formulas, an explaining text and a picture or sketch.[8] It links, for example, the deflection of a beam to the load, as well as to the geometrical and material properties of the beam itself. This specific design knowledge enables one to decide what needs to be done in order to meet the requirements. External properties can be linked to the internal properties using a mathematical, formal relation. For instance, consider the example of a profile's deflection. We already established that the profile's deflection, as an external property and is the result of a number of internal properties. Regarding Figure 3 it becomes obvious, that the deflection of a profile is not only caused by geometrical and material properties, which are internal properties of the product, but by external loads $q_0$and the location of supports. The load q_0 is the load onto the profile structure and consists of the profile's own weight but can also be an external load which is not a property of the product but a process property of the use phase. This external load is influencing the profile's deflection. Any object gets functionality and creates benefit only if it is part of a process. However, we have not yet mentioned the relationship between these parameters. If the designer wants to minimize the deflection, not only must he know which internal properties to change, but also how he should change these parameters in order to achieve increased flexural strength and therefore lower deflection.

This example of the deflection of a profile makes it clear that in order to minimize the deflection, the geometry must be changed or the modulus of elasticity must be increased. The designer can systematically change one or more internal properties in order to arrive at the desired deflection. It is also important to note that when optimizing the deflection, the modulus of elasticity cannot be changed arbitrarily. The modulus of elasticity is always a result of the selected material and is a set material parameter. This means that there is a discretely assigned value for every material. Apart from the linear flow splitting example, the concept of internal and external properties may prove as a basis for structuring design knowledge in general.

3. MATHEMATICAL OPTIMIZATION

These formalized interrelationships are used by downstream mathematical optimization processes to compute new topologies and to evaluate them to find the best solution. This topology then satisfies the necessary conditions on the product and is optimal with respect to the imposed objective function. To this end we make use of methods from mixed-integer linear programming. In the sequel we present the mathematical formulation of this model. For details of the solution method we refer to.[9] A design task is carried out as a case study to compute the topology of a two-channel profile used as a self-supported heat exchanger (e.g. hot exhaust air, cold fresh air). One main goal of the optimization is to minimize the deflection of the unit, another objective is to maximize the heat transfer between the two channels. One channel could be a hot air exhaust which should heat up cold air coming into the system to transfer as much process heat from medium 1 (hot exhaust air) to medium 2 (cold fresh air). The installation space of the entire profile is limited and parameterized by means of a polygonal shape. This constraint limits the field of solutions as a mathematical constraint.

3.1. Discretization of the Design Envelope

In general polygonal shape of a cross section area specifies the design envelope. This area is discretized by a pixel grid of quadratic pixels. The channels as well as the sheet metal have to obey this discretization. Pixels are assigned to channels, and sheet metal is placed on the edges between adjacent pixels. This assignment together is here called topology. In our example we use a topology of 6 horizontal and 6 vertical pixels, thus 36 pixels together. It is possible to fix some pixels to a certain channel and to fix some edges to sheet metal (or to fix them to remain free of sheet metal) before the automatic optimization starts. In our example we leave all decisions to the optimization algorithm. Our design task consists of 2 channels, one of size 16 pixels, and the other of size 20 pixels. Since each of the 36 pixels can be assigned to either channel, we have to deal with 2^{36} different topologies, and find the optimal one among these. Due to this enormous amount of different solutions one cannot generate all of these topologies one after the other and find the optimal one. The problem needs a systematical search for a global optimal solution, which is done by modern methods of Discrete Optimization.

Denote by V the set of pixels. In a regular rectangular grid each pixel has four neighbours. For each pair of neighbouring pixels $i, j \in V$ we introduce an edge $\{i, j\}$. The set of all edges is denoted by E. The set of channels is defined as C. For each channel $t \in C$ we assume that the cross section areas are given c^t. In our demonstrator case these are: $c^1 = 16$, $c^2 = 20$. By the topology optimization each pixel is assigned a type $t \in T := C \cup \{0\}$, where 0 represents the exterior, and the other values the respective channels.

3.2. The Basic Model

The decision which type t to assign to which pixel i is modeled by a binary decision variable $\delta_i^t \in \{0, 1\}$. For edges $\{i, j\}$ we introduce a decision variable $\mu_{i,j} \in \{0, 1\}$, that represents the decision if sheet metal is placed on this edge or not. Using these two families of variables we can state the basic topology optimization model as follows:

$$\min \sum_{\{i,j\} \in E} \mu_{i,j} \tag{1}$$

$$\text{s.t.} \quad \sum_{t \in T} \delta_i^t = 1 \qquad \forall i \in V \tag{2}$$

$$\sum_{i \in V} \delta_i^t = c^t \qquad \forall t \in C \tag{3}$$

$$\delta_i^t + \delta_j^t + \mu_{i,j} \le 2, \ \left|\delta_i^t - \delta_j^t\right| \le \mu_{i,j} \qquad \forall t \in T, \{i,j\} \in E \tag{4}$$

$$\mu_{i,j} \in \{0,1\} \qquad \forall \{i,j\} \in E \tag{5}$$

$$\delta_i^t \in \{0,1\} \qquad \forall t \in T, i \in V \tag{6}$$

By solving this basic model we are able to compute a topology of a multi-chamber conduit with given cross section areas per channel that has a minimum total weight. The objective function (1) counts the total number of sheet metal edges, and thus by is minimization light-weight topologies are preferred. Constraint (2) ensures that each pixel carries exactly one type. Constraint (3) requests that the number of pixels that are assigned to a certain channel equals the cross section area of this channel. By constraint (4), two neighboring pixels are separated by sheet metal if and only if they belong to different channels. Constraints (5) and (6) require the integrality of the decision variables.

3.3. Beam Bending Expressed as a Mathematical Objective Function

Since the length of the product is large in comparison to the width and the height, the bending can be computed using Euler-Bernoulli beam theory. This theory states that the deflection $w(x)$ of a large and thin beam satisfies the ordinary differential equation $E \cdot I \cdot \frac{d^4w}{dx^4} = q_0(x)$. Here E is the modulus of elasticity, a material dependent parameter, I is the area moment of inertia with respect to the central axis of the profile, q_0 is the load distribution function along the beam. For a beam that is simply supported at both ends one obtains from the solution of this differential equation $w_{\max} = \frac{5q_0l^4}{384EI}$ as the maximal deflection under self load, where$l$ ist the length and w is a homogeneous distributed self load of the product. In order to minimize the maximal deflection under self load we have to find a topology that minimizes the ratio $\frac{q_0}{I}$. A minimization of the deflection as an objective function is equivalent to a maximization of the stiffness. The main parameter that one can influence in this respect is the area moment of inertia. This is a result of the corresponding inner properties of the geometry.

3.4. Minimizing the Deflection

Per definition the area moment of inertia is given as $I = \iint\limits_B (s_y - y)^2 dxdy$, where s_y is the vertical coordinate of the center of mass. By B we denote the geometry of the cross section. In order to apply the methods from Discrete Optimization the area moment of inertia has to be linearized and discretized. This yields $I \approx \sum\limits_{\{i,j\} \in E} (I_{i,j} + (s_y - y_{i,j})^2 \cdot \mu_{i,j}$, where $y_{i,j}$ is the vertical coordinate of the center of mass of sheet metal on edge $\{i,j\}$ and $I_{i,j}$ is the area moment of inertia of edge $\{i,j\}$. The latter can be evaluated exactly in the following way: If $\{i,j\}$ is a horizontal edge, we obtain $I_{i,j} = \frac{hb^3}{12}$, and for a vertical edge we get $I_{i,j} = \frac{bh^3}{12}$, where b is the thickness of the sheet metal and h is the width resp. height of a (quadratic) pixel.

The center of mass (s_x, s_y) can be placed by the designer anywhere within the design envelope. To ensure that the center of mass of the topology is actually at this specified point, the following constraint has to be added to the model:

$$-\varepsilon \leq \sum_{\{i,j\} \in E} (s_x - x_{i,j}) \cdot \mu_{i,j} \leq \varepsilon, \quad -\varepsilon \leq \sum_{\{i,j\} \in E} (s_y - y_{i,j}) \cdot \mu_{i,j} \leq \varepsilon \tag{7}$$

The (small) value ε defines the discretization error.

3.5. Connectivity Constraints

Moreover it is necessary to explicitly integrate connectivity constraints for the channels into the model. That means, is it not allowed to place some parts of the channel in one corner of the design envelope and the remaining part in the other corner without any connection between these two parts. Connectivity can be formulated in various ways. One way, the so called flow formulation, was presented in Ref. 10 In the sequel, we present another method, the so called cut formulation. A computational comparison between them can be found in Ref. 1.

Let $i,j \in V$, $i \neq j$, be two non-neighboring pixels. A node cut $N(i,j) \subset V$ is a subset of nodes with the property that i is on one and j is on the other side of the cut. In this sense the cut separates i from j. If pixel iN is assigned to channel t and pixel j is also assigned to channel t then for every node cut $^{N(i,j)}$some pixel from this cut has also to be assigned to channel t. This constraint can be formulated as the following linear inequality:

$$\delta_i^t + \delta_j^t \leq 1 + \sum_{k \in N(i,j)} \delta_k^t. \tag{8}$$

3.6. Computational Results

Besides a minimal deflection we aim to study the heat transfer behavior of the topology. In our simplified physical model we assume as a first approximation, that the heat transfer between the fluids in the channels is to be proportional to the surface area between the two channels. This in itself is proportional to the length of the sheet metal separating the two channels in the cross section area. However, an increased length comes at the price of a higher weight of the product. In order to obtain optimal solution for the model we apply a branch-and-cut method. During the solution finding process sub-optimal solutions are generated. In such solution all constraints are satisfied, but the area moment of inertia and therefore the deflection is not optimal. In Figure 5 we show the solution space of 2^{36}feasible and non-feasible solutions. This solution field gets limited by constraints. Every solution that does not fulfill one or more constraints is a non-feasible solution (e.g. a profile with 3 channels). Constraints include feasible and exclude non-feasible solutions in the entire solution space. Objectives and wishes allow one to rank the remaining solutions in regard to their performance. One optimal solution for every objective ("min. mass", "min. deflection" and "max. heat transfer") has been computed. But the optimal solutions computed have conflicting results. The optimal solution with the best heat transfer in between channel 1 and 2 has the highest mass in the field of feasible solutions. For that reason required external properties have to be weighted and the computed topologies can be evaluated according to weighting factors so that one topology can be found that suits the best. If we have conflicting external properties and therefore conflicting objective functions our computed topology will always be a compromise in between the optimal solutions. We can compute a feasible solution that is a compromise in between all optimal solutions.

4. DISCUSSION AND PROSPECTS

We have shown that the development process of simple profile structures can be algorithmized to a large extent. Design knowledge and therefore the knowledge of properties and their relations is a prerequisite for an algorithm-based search for an optimal topology, to limit the solution space and evaluate solutions. The topologies determined in this process are still not finished designs, but they

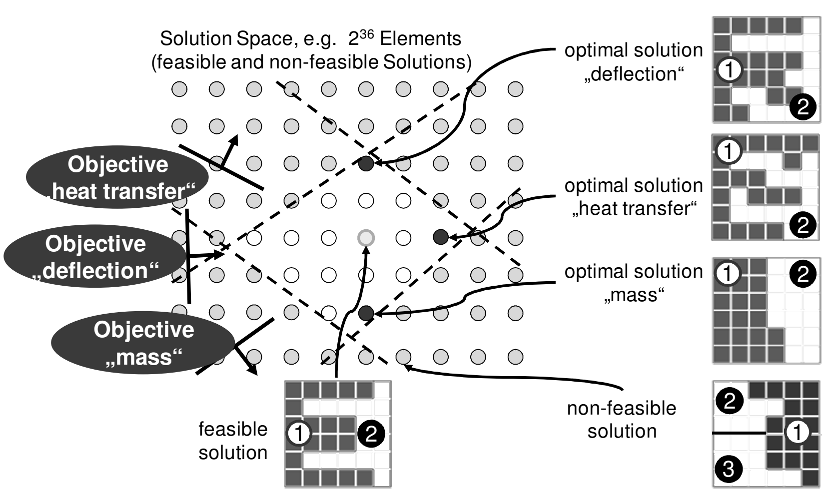

Figure 5. Solution space with non-feasible, sub-optimal and global optimal solutions.

supply the designer with valuable suggestions, as to the possible form of a topology which is optimized to certain required external properties. The topology optimization is therefore embedded into further steps of calculation in CRC 666. In a subsequent optimization step, a more detailed product geometry should be obtained by solving a non-linear continuous shape optimization model. The geometry of the wall thickness is optimized to optimize the objective function and correspond to the constraints of the manufacturing processes. Finally, in a third step, restrictions of manufacturing processes are taken into account. Given a profile with a functionally optimized product geometry, it must still be determined how it can be manufactured using a linear flow splitting process. Our ongoing research concentrates on an inclusion of further engineering constraints and objectives, such as fluid dynamics or torsion and a more elaborate description of properties and their relationships to each other in the form of an investigation of more physical models, tables, diagrams, texts etc. An entire collection of rules is the basis for further optimization processes.

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