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ANALYSIS OF THE TOOTH ROOT STRESS IN HIGH TRANSVERSE CONTACT RATIO INVOLUTE GEARING

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Keywords: high transverse contact ratio, involute gearing, stress, tooth root

1. Introduction

High transverse contact ratio gears (abbr. HCR-gears), Fig. 1, are non-standard involute spur gears with a transverse contact ratio $e_\alpha > 2$, which can be obtained by increasing the gear tooth height and/or by diminishing the normal pressure angle a_n. This type of gears has double and triple teeth pair contacts alternatively during the mesh, i.e. every accurately machined and absolutely rigid tooth should be hypothetically loaded only by alternatively 1/2 and 1/3 of the total normal force F_{bt} - dashed lines in Fig. 2. This feature has as a consequence bigger load capacity, and that is the reason why HCR-gears have an increasing application in passenger cars (e.g. Peugeot, Citroen, Škoda, Opel etc.), trucks, aircrafts and ships [Villars 1999, Munro 1994]. Other advantages of the HCR-gears are more uniform torque transmission and noise reduction, also of importance in vehicles. The most important drawback of the HCR-gears is an increased sliding friction power loss, although it can be minimized by proper choise of the geometrical tooth parameters [Križan 1998, Križan 1999].

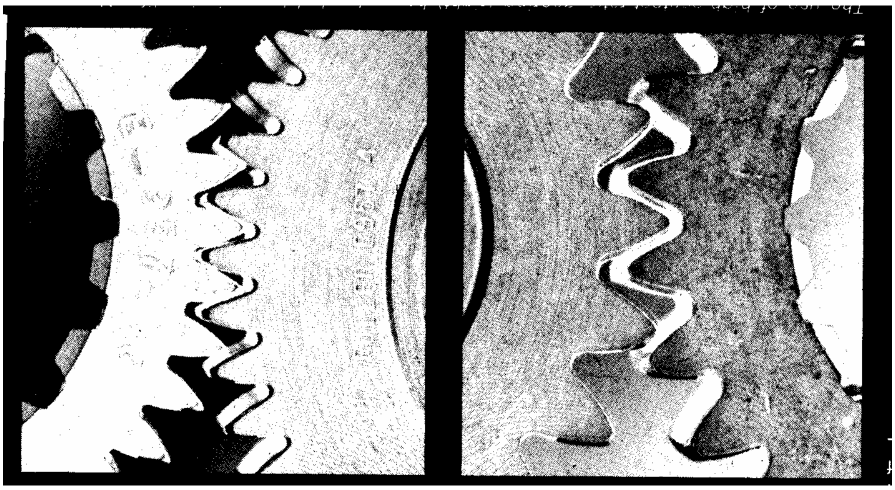

Figure 1. HCR gearing - left; Standard involute gearing - right

2. Load distribution on tooth pairs in mesh

As the teeth in reality are not rigid but somewhat elastic, the real load distribution during the mesh will differ from the one presented by the dashed lines in Fig. 2. To carry out a precise calculation of the real forces in HCR-gears, a precise method based on teeth elastic deformations has been developed [Lovrin 2001]. This method also enables the determination of the real tooth root stresses on a series of gear pairs. In this paper three HCR-gear pair examples with following geometrical parameters will be presented (subscript 1 = pinion, 2 = wheel):

Example 1: normal pressure angle a_n= 20°; module m_n = 22 mm; tooth addendum factors $h_a{}^*{}_{0\ 1,2}$ = 1,5; tip radius of the cutter tool factors $r_a{}^*{}_{0\ 1,2}$ = 0,341; bottom clearance factors $c^*_{1,2}$ = 0,1; model thickness (gear width) $b_{1,2}$ = 10 mm; teeth numbers $z_{1,2}$ = 22/88; addendum modification coefficients $x_{1,2}$ = 0; transverse contact ratio e_α = 2,3; total tangential force F_t = 11840 N.

Example 2: a_n= 16°; m_n = 12 mm; $h_a{}^*{}_{0\ 1,2}$ = 1,5; $r_a{}^*{}_{0\ 1,2}$ = 0,25/0,1; $c^*_{1,2}$ = 0,1;
$b_{1,2}$ = 10 mm; $z_{1,2}$ = 35/90; $x_{1,2}$ = +0,3/-0,3; e_α = 2,64; F_t = 6825 N.

Example 3: a_n= 14,5°; m_n = 17 mm; $h_a{}^*{}_{0\ 1,2}$ = 1,6; $r_a{}^*{}_{0\ 1,2}$ = 0,2; $c^*_{1,2}$ = 0,25;
$b_{1,2}$ = 10 mm; $z_{1,2}$ = 50/100; $x_{1,2}$ = 0; e_α = 2,88; F_t = 4494 N.

Realistic load distributions for examples 1, 2 and 3 during the mesh are shown in Fig. 2: the maximal normal force F_{bti} on a tooth along the path of contact A-G exceeds in all theree cases 50% of the total normal force F_{bt} between the pinion and wheel. In some other cases, depending on the actual toothing geometry and manufacturing accuracy, the force acting on the tooth can be even bigger, up to approximately 70% of the total force.

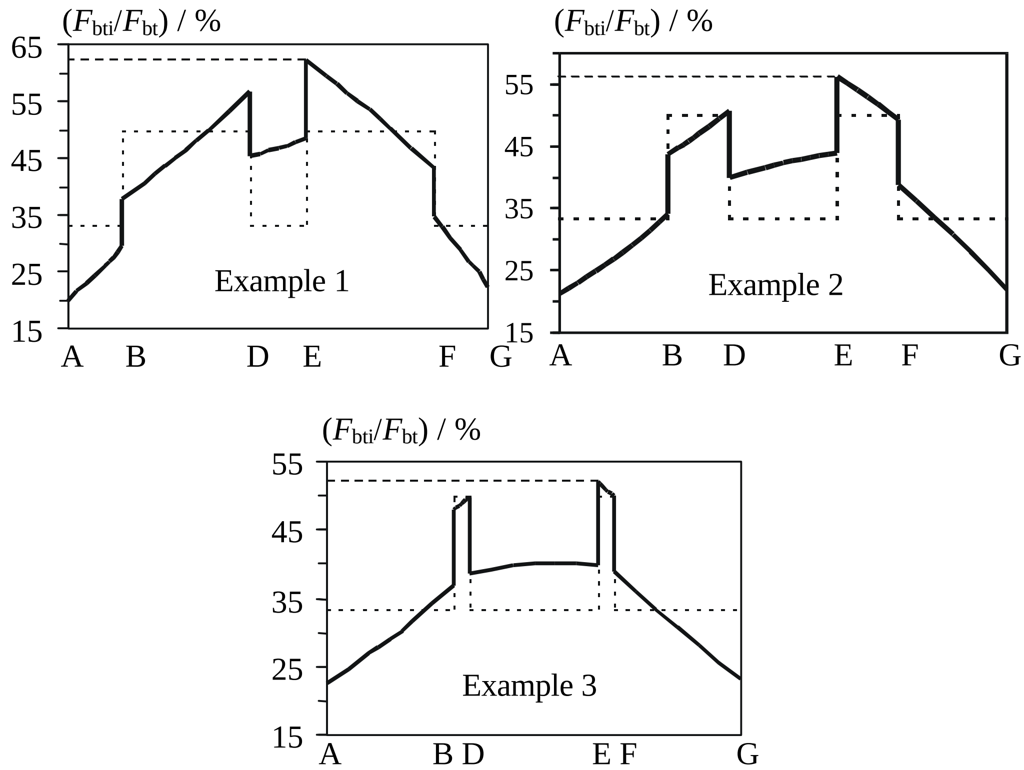

Figure 2. Realistic load distribution on HCR-gears teeth along the meshing line for examples 1, 2 and 3

3. Analysis of the tooth root stress in HCR gearing

Tooth root stress along the path of contact, for both pinion and wheel, was calculated using the equation

$$s_{\mathrm{F0i1,2}} = \frac{F_{\mathrm{ti}}}{b \cdot m_{\mathrm{n}}} \cdot Y_{\mathrm{FSi1,2}} \tag{1}$$

where Fti is the realistic tangential force (acc. to Figure 2) and YFSi is the effective tooth form factor in the corresponding point i along the path of contact A-G.
Tooth root stresses in the corresponding points (D for pinion and E for wheel) according to [DIN 3990 1987], were calculated using the equation

$$s_{\mathrm{F01,2}} = \frac{F_{\mathrm{t}}}{b \cdot m_{\mathrm{n}}} \cdot Y_{\mathrm{FS(D,E)}} \tag{2}$$

where Ft is the total tangential force and YFS(D,E) is the effective tooth form factor in the corresponding points D - for pinion and E - for wheel.
It must be emphasized that the performed calculations are based on a method that differs from the usual DIN-method [DIN 3990 1987]. The DIN-method deals with stresses in the tooth root fillet point, where the tangent makes an angle of 30° with the tooth centerline. This is not quite correct, because the maximum stress point, in most cases, deviates from this point. The method used for tooth root stress calculations presented in this paper deals with the points where maximal stresses really occur [Obsieger 1980].
The Figures 3, 4 and 5 show the calculated tooth root stresses along the meshing line for pinions and wheels in examples 1, 2 and 3. Also presented are the tooth root stresses for the pinion at the point D and the wheel at the point E calculated according to the DIN-Standard procedure [DIN 3990 1987].

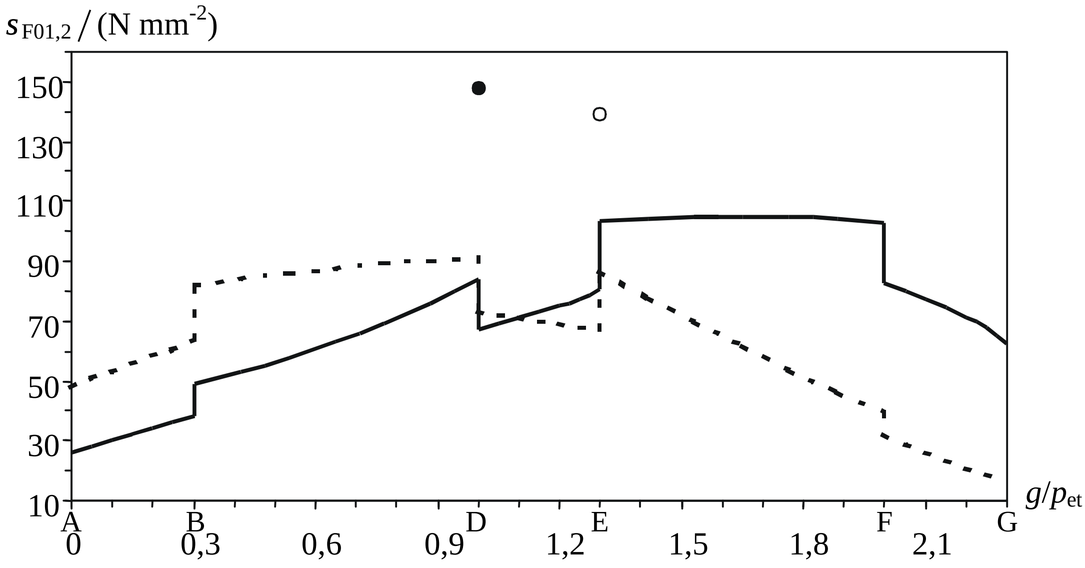

Figure 3. Example 1: tooth root stresses along the meshing line for pinion and wheel (——— pinion, – – – · wheel, ? pinion [DIN 3990 1987], ? wheel [DIN 3990 1987])

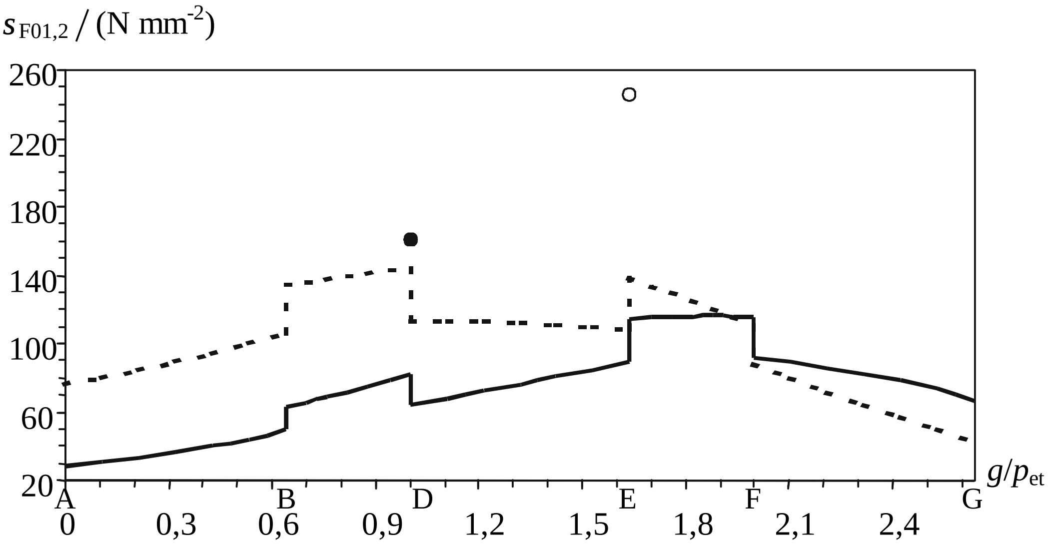

Figure 4. Example 2: tooth root stresses along the meshing line
(——— pinion, – – – ·wheel, ? pinion [DIN 3990 1987], ? wheel [DIN 3990 1987]))

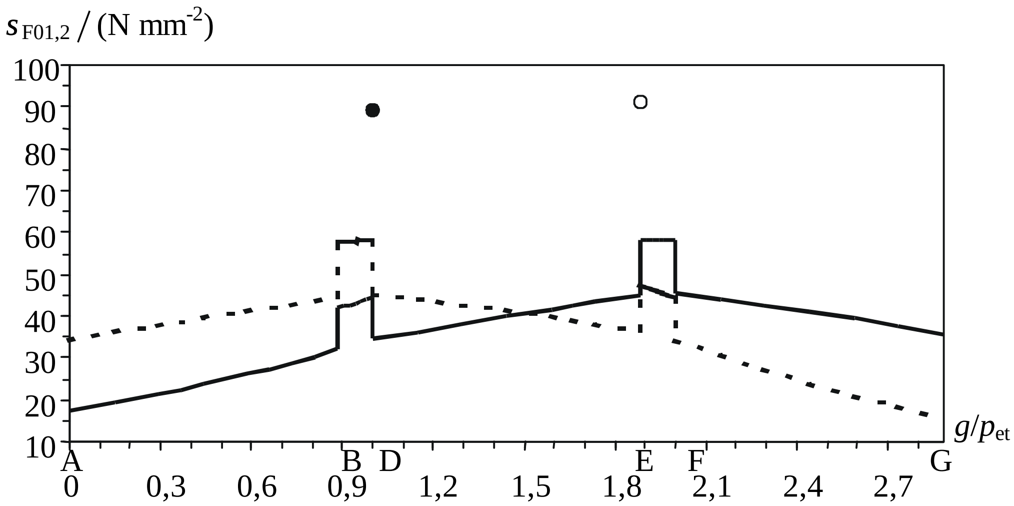

Figure 5. Example 3: tooth root stresses along the meshing line
(——— pinion, – – – ·wheel, ? pinion [DIN 3990 1987], ? wheel [DIN 3990 1987])

The analysis of the tooth root stresses in HCR involute gearing shows that the stresses are usually rather smaller than the results obtained according to [DIN 3990 1987]. The accuracy of the results presented by lines in Fig. 3, 4 and 5 have been proved experimentally using photoelastic gear models in special loading frames [Lovrin 2000].

This means that the DIN-Standard calculation gives rather inaccurate results, although they are "on the safe side". This follows from DIN-Standard HCR-gears calculation procedure being based on the following approximations:

- It deals with stresses in the tooth root fillet point where the tangent makes an angle of 30° with the tooth centerline.
- The calculation of stresses is carried out with the total force acting on the tooth, although theoretically it can be at the most 1/2 of the total force Fbt acting on the distance hF ADP from the tooth root, Fig 6. This method has been accepted in order to enable the use of standard formulae for standard toothing with $\varepsilon\alpha < 2$.
- To compensate for this intentional error, it is assumed that the total force Fbt acts on much shorter distance hF IDP from the tooth root than it is the case with standard gears.

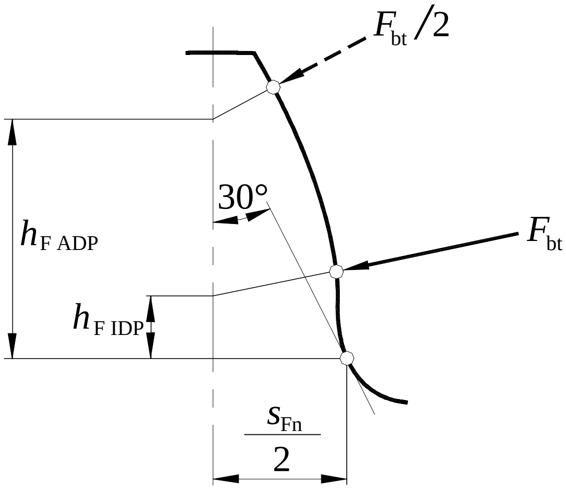

Figure 6. Decisive meshing point for tooth root stress in HCR gearing [**DIN 3990 1987**]

4. Conclusion

A comparative analysis of the tooth root stress in high transverse contact ratio involute gearing was made. A comparison of the results obtained by this procedure [Lovrin 2001] with the values that are obtained by using the DIN-Standard HCR-gears calculation, shows that DIN-Standard procedure gives too intense stresses in the tooth root, which means that these gears will be oversized.

The method developed in [Lovrin 2001] for a more precise stress determination, using load distribution on gear tooth pairs in mesh calculated on the basis of their elastic deformations, enables a more reliable calculation of HCR-gears load capacity. HCR-gearboxes designed by use of the presented improved calculation will not have any unnecessary safety reserve, they will be smaller and lighter, and consequently the friction loss will also be smaller as it would be the case when using the DIN-calculation.

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