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VIRTUAL ASSEMBLY ANALYSIS: STANDARD TOLERANCE ANALYSIS COMPARED TO MANUFACTURING SIMULATION AND RELATIVE POSITIONING

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1. Introduction

Whenever parts are manufactured, they differ from their intended ideal shape. Therefore, functional and/or aesthetic problems may occur. In the detailed design phase, the product developer defines tolerances to limit acceptable geometric errors. Large tolerances can cause mounting problems, visible variations of gap and flush between components (low quality appearance) and operating problems (high energy consumption, high abrasion). Close tolerances necessitate expensive manufacturing methods and have to be verified with time-consuming measurements. To find a compromise and to prevent severe loss of profit, tolerance analysis methods are widely used. These methods can be divided into manual two-dimensional calculations and computer-supported three-dimensional stochastic simulations. The main drawback of both methods is their high level of abstraction: Specific manufacturing deviations (e.g. springback, mold shrinkage) which depend on the production method and part geometry are not taken into account in the analysis. The assembly process is not reproduced, but extensively simplified. Therefore, it is not possible to visualize assembly variants which include shape deviations; the results are presented as distribution curves of two-point or feature based measurements.

We propose to include realistic manufacturing deviations of parts by means of Finite Element Analysis and to perform a geometrically correct simulation of part assembly through Relative Positioning. Benefits of the approach are: The simulation is less abstract, more comprehensible and therefore less error-prone than commonly used methods. A final visualisation of the results can reveal the areas of high geometric variation. Therefore, this approach contributes to the improvement of virtual quality assessment during product development: decision making on functional requirements is improved because more realistic stochastic deviations can be visualized and analyzed. Moreover, this information can be used to optimize assembly inspection in the production phase because critical quality measures can be identified and thus the total amount of measures can be reduced significantly.

The paper is structured as follows: First, the proposed work methodology is explained. The steps of the methodology are compared with state-of-the-art tools and methods of statistical tolerancing. In Section 3, a detailed case study of an assembly consisting of two stamped parts is described. The procedure and results of state-of-the-art tolerance analysis are compared to our methodology and post processing, especially visualisation techniques. Section 4 concludes our work and gives an outlook of possible future extensions.

2. State-of-the-art compared to Virtual Assembly Analysis

This section compares the chosen work methodology to the state-of-the-art simulation process using Monte-Carlo simulation implemented in commercial computer aided tolerancing (CAT) software.

2.1 Monte-Carlo simulation

Due to the fact that manufactured parts differ from their intended ideal shape, a way has to be established to represent non-ideal geometry for simulation. Therefore, a feature- or point-based representation of parts was chosen in (CAT) simulation packages in order to generate deviating parts according to their specification limits and assigned distribution characteristics [Wisniewski,1998]. Though several different approaches such as Taguchi´s Method [Nigam,1995] or Second-Order Tolerance Analysis exist for numerical evaluation of the assembly, the Monte-Carlo technique is widely used in CAT-software packages. The simulation process can be subdivided into the following steps [Wisniewski,1998] (see Figure 2a): **Pre-processing** includes the specification of parts geometry based on features $FEAT_i$ (points, planes, pins, holes, slots, tabs…) and assembly operations $ASMOP_j$, the definition of tolerances T_k and the specification of quality measures PKC_m (Product Key Characteristic). A **simulation of deviating geometry** and assembly is carried out by randomly selecting the feature location and orientation of $FEAT_i$ according to the tolerance specification T_k and the selected distribution type by using the plain Monte-Carlo technique. A re-assembling based on the previously generated sample of features according to the modelled assembly operations $ASMOP_j$ leads to the computation of the results for quality criteria PKC_m. The **post-processing** of the simulation results yields information on measurement points PKC_m (distributions, mean, standard deviation, process capability indices) and sensitivity measures or a contributor report. The major advantage of this procedure is that the functional behaviour is described by analytical equations, which means evaluation using the Monte-Carlo method can be performed fast. The sensitivities and contributor reports obtained are important tools for the designer to improve the tolerance design. The Monte-Carlo simulation process is outlined in Figure 2a.

This method also incorporates some important drawbacks. Due to the analytic problem formulation only ideal geometry features can be modelled. Therefore, interactions of parts with a small ratio of mating areas between parts surfaces cannot be modelled physically correctly. A local shape deviation, which could be a characteristic of a certain manufacturing process such as bending, for example, cannot be captured. Moreover, the quality of the result depends on the correct selection of PKC_m-measurement locations, which means that critical regions have to be anticipated by the user. This is a crucial task which can lead to incorrect interpretations of the problem.

2.2 Virtual assembly analysis: Generation of non-ideal parts and virtual part assembly

Because of these drawbacks, an alternative way to model the deviations is chosen in this paper. Instead of a feature-based generation of deviating part geometry a mesh-based model is used. Finite Element Analysis (FEA) is used to obtain realistic shape and orientation deviations caused by the natural scatter of process parameters in manufacturing processes (see Figure 2b). The process of sheet metal forming is selected as manufacturing process in this work because of the following reasons:

- availability of powerful simulation tools employing accurate material models and taking into account multistage process steps (gravity, holding, stamping, springback) [Jansson2007],
- highly geometry-dependent process and thus non- or hardly-transferable geometric deviations such as springback [Roll2005] and thickness distributions [Jansson, 2007],
- similar order of magnitude of simulated deviation ranges and tolerances [Jansson, 2007].

Stamping is a process which is liable for a multitude of influences. The use of simulation software permits an analysis of the metal forming process taking the scatter of its crucial process parameters into account. In general these parameters can be subdivided into a plenty of material-, workpiece- and process-related variables. But direct Monte-Carlo simulation with this amount of parameters is computationally expensive. Therefore, a different strategy is chosen: Starting with a selection of crucial process parameters and their stochastic behaviour a sampling of scattering input parameters is performed. The combination of scattering input parameter sets is used afterwards for modification of

the stamping simulation input in order to obtain a stochastic response. The selection of pivotal process parameters is important to reduce the amount of input variables because they determine the amount of necessary FEA-simulation runs which are computationally expensive. These variables can be identified by performing a sensitivity analysis or based on the experience of manufacturing specialists. The suitable probability distributions of these parameters have to be selected properly depending on the problem formulation. This can be achieved by the use of quality management data of the sheet metal and the manufacturing process. Otherwise, an estimation of the four statistical moments has to be performed. This information is used to generate input parameter combinations, which take account for the whole range of the parameter scatter and the involved distributions: This can be achieved by dividing the probability distribution into n sections of equal probability. Within these subsections an analysis point is selected randomly. Every input variable is mapped to a vector with n discrete input values resembling the specified distribution. All input parameter vectors are joined into an n x n matrix after permuting the vector components in such a way that a great area in the whole input parameter space is covered. The n rows of the matrix then contain n simulation input parameter sets for use in stamping simulation. This approach is called Latin Hypercube Sampling [McWilliams, 1986],[Moshfeg, 2008]. Compared to the Monte-Carlo analysis, the amount of required simulations n is reduced significantly by this method. Major advantage of this approach is that the admissible range of scatter is exhausted and no part of the probability distribution is left out.
To build up simulations of the manufacturing process, the FEA-Package PamStamp 2G™ is used. An initial model of the stamping process is set up. Each parameter set gained by Latin Hypercube Sampling is applied to the model. It allows performing n simulations leading to n deviating geometries of the stamped parts. This geometry is exported as a triangle mesh of the simulated part, including deviations and sheet thickness.
All meshes produced by the stochastic FEA differ from their ideal CAD shape. Therefore the joined surfaces of the assembly's components do not fit together in a perfect way. The added non-ideal part may float over the surrounding geometry or it may collide (see Figure 1). Thus, the mounted part has to be moved relative to its surrounding geometry. This process is called Relative Positioning. Rigid parts possess six degrees of freedom (three translational, three rotational). Therefore, the Relative Positioning can be formulated as the determination of meaningful parameters for the six degrees of freedom. For an accurate positioning, the problem is too complex to perform an exhaustive search and features many local minima. For this reason, we have developed a framework which includes several heuristic optimization algorithms (Simulated Annealing, Evolutionary Strategy and Particle Swarm Optimization). Depending on the optimization strategy, the algorithm scans the search space and rates 6D-positions. When a predefined termination criterion is reached (the maximum number of position ratings) the algorithm returns the position with the best found rating.

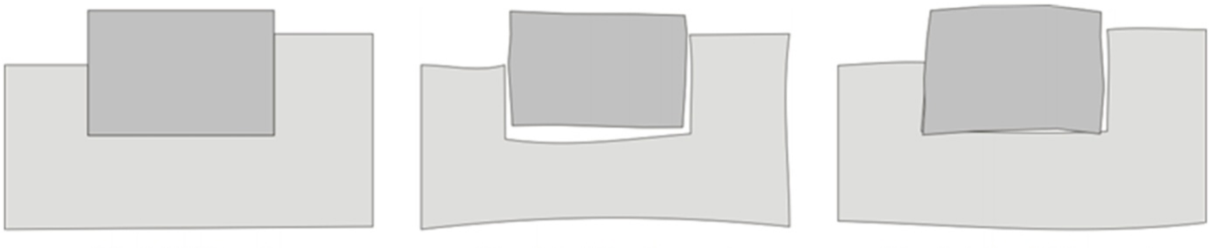

Figure 1. Possible positioning problems of deviating parts

The part positions are rated by building a weighted sum over several functions. These functions are also called "cost functions". Better positions are rated with lower costs, so the optimizer has to find the global minimum. In the case study (Section 4), the following two cost functions were used: **Collision Avoidance:** All triangles of the positioned part are checked for collision or interference with the surrounding geometry. If any collision is found, a high constant penalty value, else 0 is returned. To speed up the calculation, hierarchical collision detection was used.

Direction Rating: To simulate a realistic assembly process of physical parts, a direction vector is defined. The cost function rates how far the part can be moved into the direction of the given vector. Far movements are rated better. This new approach allows for the virtual reproduction of a 3-2-1 part positioning scheme, which is important for industrial applications.
Through the combination of cost functions, a wanted assembly operation can be modelled. Example: By combining Direction Rating and Collision Avoidance, the part is pushed as far as possible in the defined direction without colliding with the surrounding geometry. Additional cost functions have been implemented for different operation purposes, see [Gnezdilov, 2009].
The assembly process is not limited to two parts: An (already simulated) subassembly is iteratively complemented by an additional non-ideal part until the whole assembly is completed.

2.3 Interpretation of the simulation results and comparison

The virtual manufactured assembly analysis yields numerous non-ideal assemblies. Each assembly consists of positioned, intersection-free part variants, represented as fine triangle meshes. The meshes allow the use of visualisation methods and measurements that take into account "real" geometry. After the inspection of simulation results, additional measurements can be defined. The results are intuitively understandable for the product developer, they show possible variants of the designed product. The product developer can visually check whether the assembly simulation has produced meaningful assemblies and gains insight in the effects of realistic geometric deviations.
In contrast, the commonly used Monte-Carlo simulation reduces the geometry to abstract representation elements (i.e. vectors). The user has to define measurements (i.e. Point-to-Point) before starting the simulation. The results are presented as distribution and statistical values. A meaningful geometric representation of non-ideal assemblies cannot be obtained; therefore, the product developer has to trust the correctness of a complex and abstract CAT model.
In contrast to the state-of-the-art Monte-Carlo based CAT approach, our work methodology can be summed up as follows (see Figure 2b, c). Whereas in commercial CAT-packages sample generation is performed on basis of Monte-Carlo and ideal mating features (see Figure 2a), the stochastic manufacturing simulation serves as a basis for creating a set of non-nominal geometries (see Figure 2b). These geometries are handled afterwards by a positioning simulation process to generate samples of assemblies (see Figure 2c). Visualisation methods and inspection tools serve as a basis for dimensional evaluation of the part and assembly geometry.

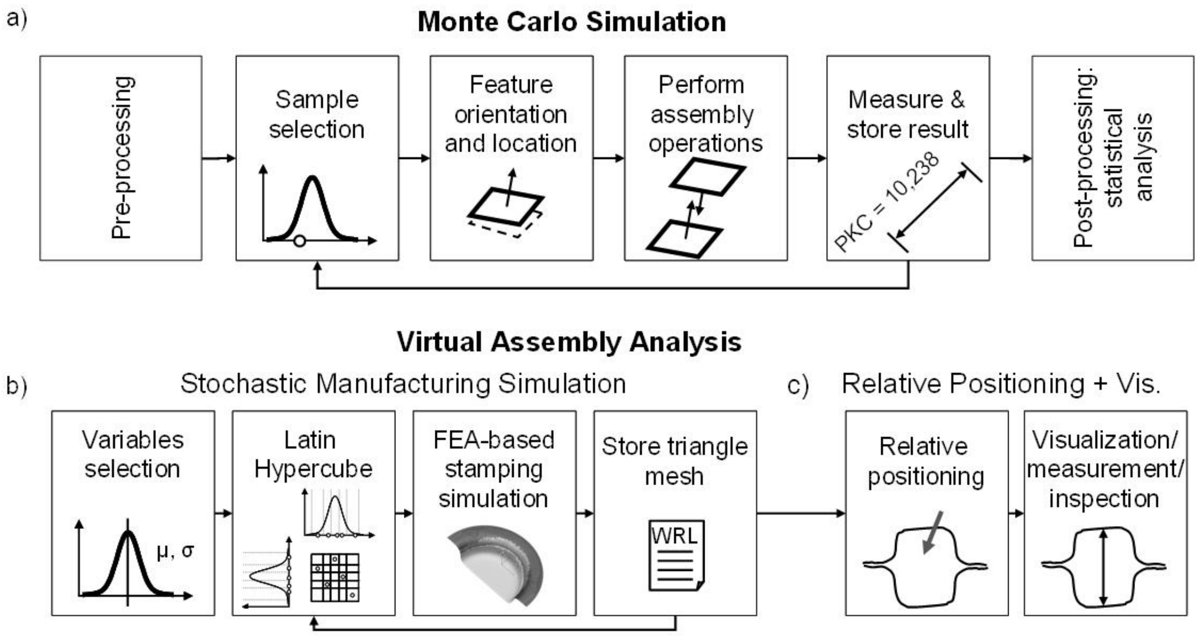

Figure 2. Comparison of Monte-Carlo method for CAT-analysis to the presented approach

3. Scientific case study

3.1 Problem definition and tolerance synthesis

During the development of an automotive car body, an important task is the definition of the dimensional requirements. The requirements have their origin, for example, from the rangeability of a sealing between car body and door. The specification of the sealing reveals that a working range of 6 mm is available. Some ratio of the range is consumed by external loads during product use (wind, temperature). For manufacturing deviations of the car body, a tolerance of ± 1.5 mm remains (DF1, see Figure 3). The requirement M3 of the subassembly of the B-pillar is crucial for the measure DF1 and it is furthermore dependent on a series of properties: PKC P1, PKC P2 and part properties of the assembly. For reduction of complexity of the case study the B-pillar cross section is selected in order to illustrate the approach. It is represented by two simplified, cross-shaped parts (see Figure 4).

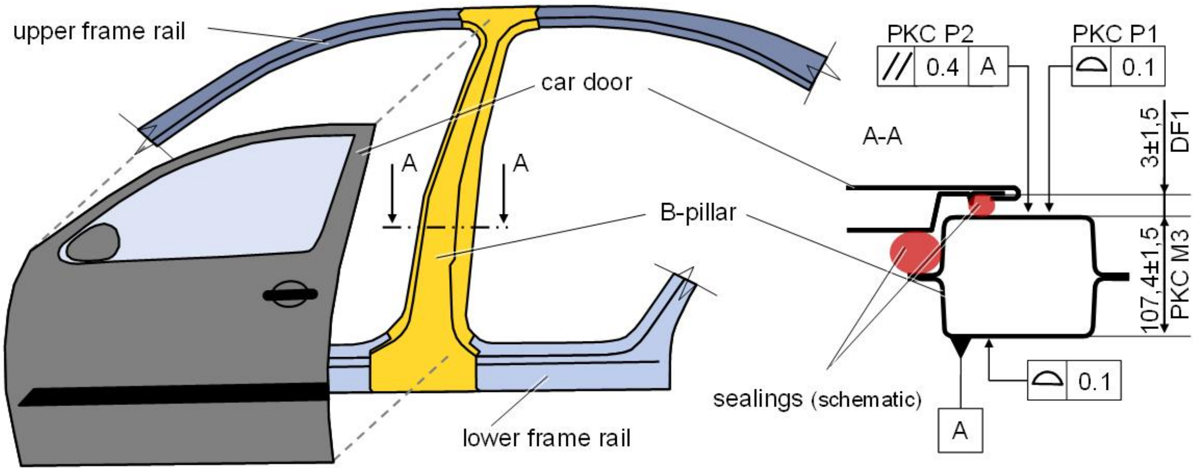

Figure 3. Functional dimensional requirements on the assembly

In product development phase the cross section A-A requires appropriate tolerance specification. To derive suitable tolerances for the individual parts of the assembly, tolerance synthesis is used [Jorden, 2001]. The basic criterion for deriving the tolerance values is the dimensional requirement M3. It serves as the target value for an analytic setup of the functional relationship (see Figure 4).

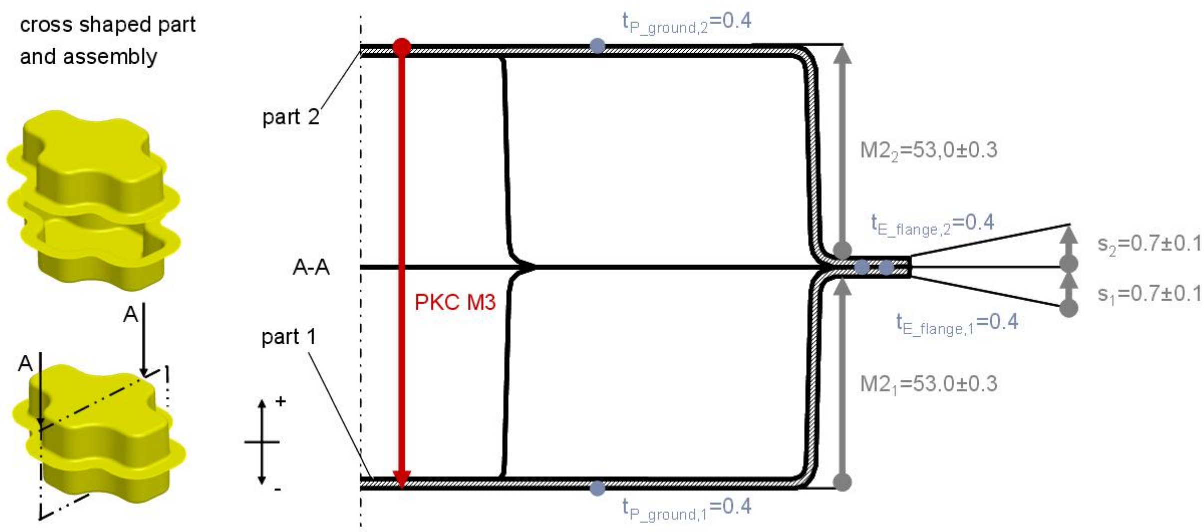

Figure 4. Functional relationship for tolerance synthesis

The components of the functional relationship, which cause a change in M3, are the parallelism tolerances $t_{P\_ground,i}$, the flatness tolerances $t_{E\_flange,i}$ and the dimensions $M2_i$ and s_i (i = 1 or 2). The direction of these components is chosen in positive direction because their change increases M3. Other tolerances on the part do not contribute to the measure M3. The following equation can be derived resembling the basic functional relationship.

$$M3 = t_{P\_ground,1} + M2_1 + t_{E\_flange,1} + s_1 + s_2 + t_{E\_flange,2} + M2_2 + t_{P\_ground,2} \tag{1}$$

The tolerance values for the components of the equation are chosen from ISO 2768-mK to serve as a basis for tolerance allocation to meet the requirement of ±1,1 for M3. This choice results in target value $M3_N$, maximum worst case $M3_u$ and minimum worst case $M3_l$, the tolerance range T_{M3_a} and dimensional specification for M3 of

$$\begin{aligned} &M3_N = 107.4\,mm \quad M3_u = 109.8\,mm \quad M3_l = 106.6\,mm \\ &T_{M3\_a} = M3_u - M3_l = 3.2\,mm \quad \Rightarrow M3 = 107.4^{+2.4}_{-0.8}\,mm \end{aligned} \tag{2}$$

This data is used to derive the statistic measures T_{M3_q} and T_{M3_r} which serve as estimates for the resulting deviation range T_{M3_w}:

$$\begin{aligned} &T_{M3\_q} = \sqrt{\sum_{i=1}^{8} T_i^2} = 1.2\,mm \quad \wedge \quad T_{M3\_r} = \sqrt{3}T_{M3\_q} \approx 2.08\,mm \\ &\Rightarrow T_{M3\_a} > T_{M3\_r} > T_{M3\_w} \geq T_{M3\_q} \Rightarrow T_{M3\_w} := 1.60\,mm \end{aligned} \tag{3}$$

The value of T_{M3_w} is chosen within the interval $[T_{M3\_r};T_{M3\_q}]$ at 1.60 mm under the following assumptions: independence of the contributors, normal distribution of the deviations within tolerance ranges, no de-centering and constant distribution parameters. Compared to the specification requirements the tolerances can be increased by the factor v = 1.375 in order to decrease manufacturing costs. In contrast to the calculation result, the increase of the tolerances is limited due to technological aspects. Profile tolerances for the contact surface between sealing and B-pillar for example must be within specified values depending on sealing characteristics. Tolerances T_{gi} are defined as: $t_{gP\_ground,1}$=0.4, $M2_{g1}$=53.0±0.4, s_{g1}=0.7±0.2, $t_{gE\_flange,1}$=0.3, $t_{gE\_flange,2}$=0.3 , s_{g2}=0.7±0.2, $M2_{g2}$=53.0±0.4, $t_{gP\_ground,2}$=0.4. (see Figure 4)
The expected tolerance value T_{M3_w} for the quality characteristic M3 can be estimated using equation 1, 2 and 3 with values T_{gi} resulting in a tolerance T'_{M3_w} = 1.9 mm. The requirement of $T_{M3\_w} \leq 2.2$ mm is therefore achieved for the assembly with high probability because of the over-estimation of T'_{M3_r} = 2.5 mm in contrast to T'_{M3_q} = 1.4 mm. These values serve – applied to the technical drawing – as a basis of system analysis and optimisation using CAT-Systems. These specifications are used for comparison to a) CAT-Results and b) assembly deviations obtained from positioning simulation of deviating parts.

3.2 Three-dimensional tolerance analysis using CAT

A three-dimensional CAT-simulation is set up using ideal CAD geometry information in combination with tolerances specified on the features. It is important to mention that, in general, no information on the real ranges or the distributions of the specified tolerances is available. Especially in processes where new technologies or materials are employed, no information on these statistic parameters is available, which are pivotal for the tolerance analysis results [Stockinger, 2009].
Starting from geometry import, features are derived, tolerances specified and applied as well as assembly operations specified (see Figure 5). The assembly process is modelled and, therefore, the geometry is placed based on a 3-2-1 degree of freedom positioning strategy on a fixture. This fixture is assumed to be free of tolerances and wear. The upper part of the assembly is mounted flange to flange and edges to pins in the same manner (3-2-1 constrained part). A statistical result can be obtained

using Monte-Carlo simulation. Moreover, a sensitivity analysis is performed, which allows for the identification of geometric amplification of tolerance effects. For the case study, the sensitivity analysis reveals that the assembly is a quite linear problem. High-Low-Median analysis allows for inspection in the contributions of the specified tolerances relating to the quality measure of interest. Main contributors to measure M3 are the dimensional tolerances $M2_1$ and $M2_2$ followed by the impact of the thickness deviations s_1 and s_2 on flanges 1 and 2. The statistical results are discussed in detail in section 3.4.

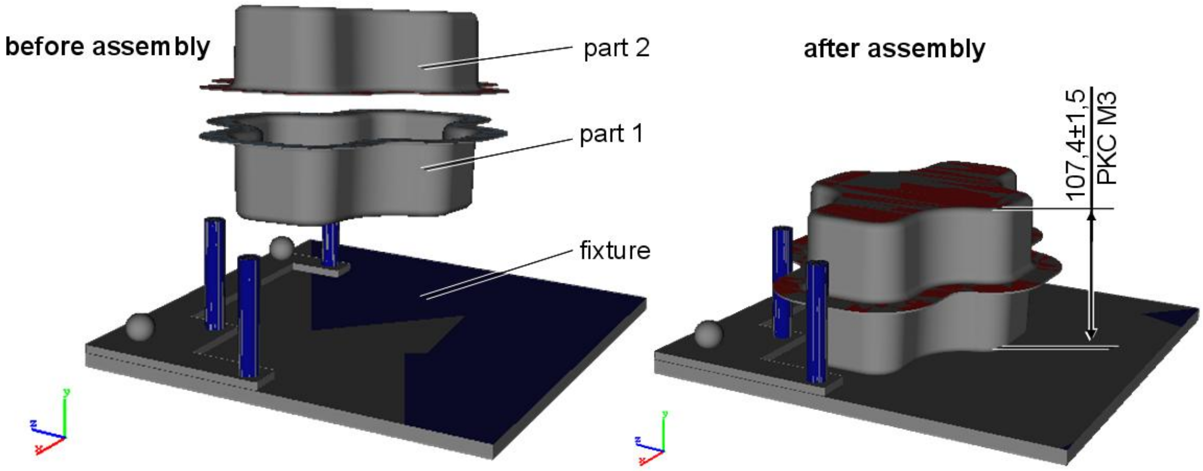

Figure 5. CAT-Model and assembly operation specifications

3.3 Three-dimensional tolerance analysis using the proposed approach

In order to simulate manufacturing, the stochastic FE simulations have to be set up. For the discretisation of the blank, shell elements are used. The tooling is modelled as rigid parts and a process macro is used to define the process including stamping velocity, blank holder force and friction. This initial model of the stamping process serves as a basis for the variation of process parameters.

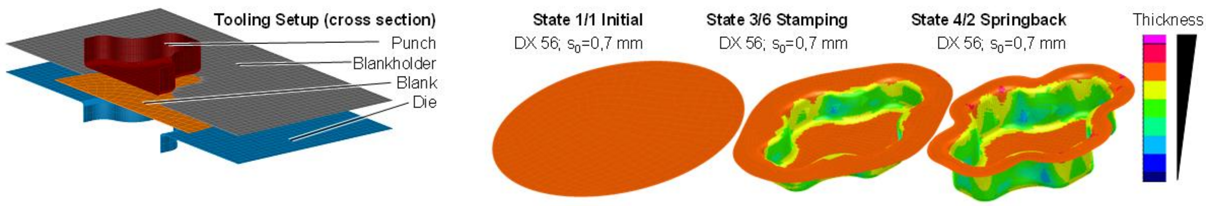

Figure 6. Stamping process setup and selected examples

The selected variables are blank thickness s_0, blank position δ_x, δ_y (due to a larger positioning device), blank holder force F_{BH}, friction μ between blank and tooling, as well as material parameters (Young's modulus E, yield strength k_f, Lankford coefficient r_0). The levels of variation are taken from in-house measurements and literature [Jansson2007], [Moshfeg2008]. These parameters are varied using Latin Hypercube Sampling under the assumption of the independence of the variables and normal distribution. The samples are generated in MATLAB using lhsnorm-function, which requires the statistic input mean and matrix of covariance (variances s_i^2) for the probabilistic measures (see Table 1). Covariance is not detected and is therefore neglected here.

An input matrix containing 74 samples is generated with this information. The sheet metal stamping simulations are performed at ~1.5 h per shot. The computational effort can be reduced from 111 h to 1.5 h using parallelisation, which is also available, for example, in automotive industry (HPC on clusters). After a conversion from mid-plane to upper- and lower boundary representation, the obtained geometry information is ready for positioning simulation.

Table 1. Input Parameter Values

	s_0 in mm	δ_x in mm	δ_y in mm	F_{BH} in kN	μ	E kN/mm²	k_f in N/mm²	r0 in -
Mean	0.7088	0	0	120	0.12	192	0	1.917
Variance	0.00006561	0.030625	0.042025	36	0.0001	25	0.00003969	0.002601

The next step is to build up assemblies consisting of two cross shaped parts. All possible combinations of parts are assembled collision-free on the positioning fixture. The main reference of the first positioned part is the ground plate of the fixture, the second reference are the two pins in –x-direction and the last reference is the pin in z-direction (see Figure 7). From an optimisation perspective, the problem was defined by two cost functions: collision avoidance between part (~66,000 triangles) and fixture (1,780 triangles) and direction rating of the part with the vector (-2, -3, 1). Each optimisation run was limited to 50,000 cost function evaluations, the average computation time for one run (positioning of the lower part) was 38 seconds. As an optimisation algorithm, Particle Swarm Optimisation was used, which performed best in previous experiments [Gnezdilov2009]. The upper part of the assembly was positioned with the same cost functions: collision avoidance between the upper part (~66,000 triangles), lower part and fixture (~66,000 + 1,780 triangles). For the direction rating, the same vector (-2, -3, 1) was used.

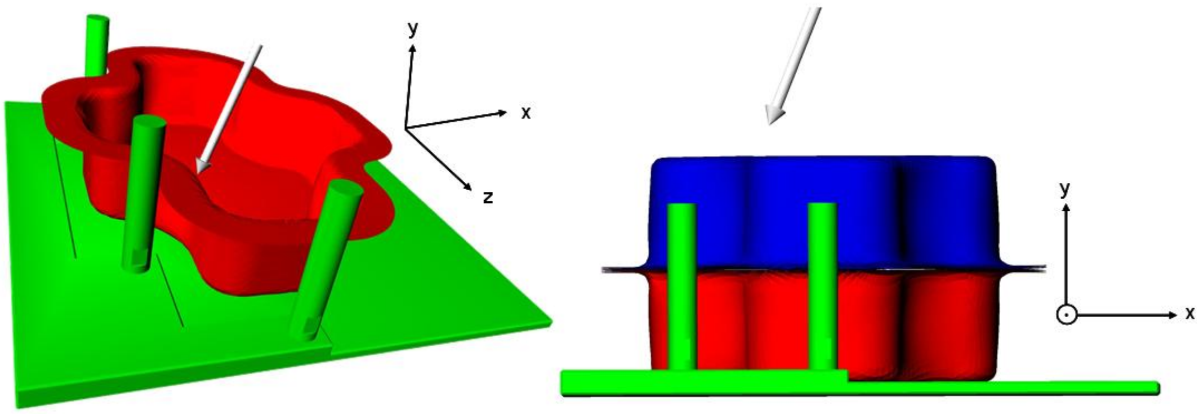

Figure 7. Positioning of the lower part of the assembly (red) on the fixture (green) and the upper part (blue) on the lower part; the white arrow shows the positioning direction

Due to the higher number of triangles involved in the collision detection calculations compared to the lower part positioning, the positioning of the upper part took 59 seconds on average. The positioning process was run on an Intel Core2 Duo CPU with 2.4GHz, two optimisations were run parallel. This reduced the runtime to approximately 50%, so all possible 74 x 74 = 5476 variants of the assembly were simulated in 45 hours. Modern CPUs with four or more cores can further reduce the computation time. The resulting assemblies were analysed with the following methods: Single part combinations can be visualized. Measurements [Penzkofer, 2008] reveal the distributions of gap distances between the parts (ranging from 0 to 2 mm, see Figure 8A), the contact surface of the flanges and the resulting height of the analysed assembly at arbitrary measurement points.

To get an overall impression of the 5,476 variants, we propose the use of volume visualization [Penzkofer2008]. Figure 8B shows the generated volume, which represents all variants in a single scene. Bright (red and orange) texture means that almost all variants are located at the same position. Volume elements coloured darkly (purple) are intersected by few part variants. The volume visualization reveals high variance in y-direction (flange and top) and low variance in x- and z-direction (side walls). Therefore the measurement M3 should be performed in the region with highest variance (marked “B”), which is an important information for quality inspection.

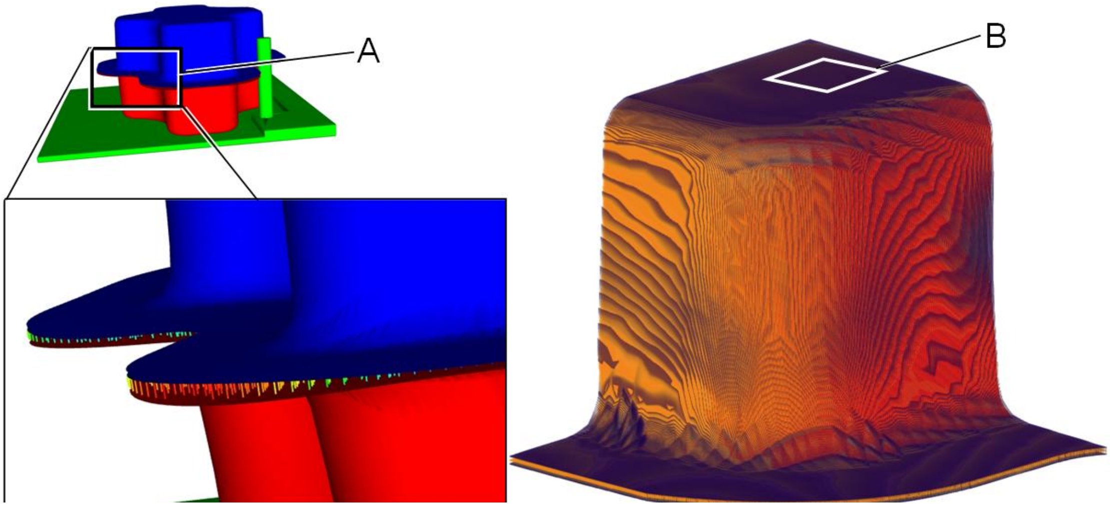

Figure 8. Gap measurement visualization for one part combination (left) and Volume Visualization of all part variants in a single scene (right)

3.4 Comparison of CAT Simulation and Virtual Assembly Analysis results

A comparison of the quality criteria height M3 of the assembly reveals an offset of the mean values of 1.35 mm: the CAT-Simulation assembly measures 107.49 mm (stddev. 0.22 mm) on average whereas Virtual Assembly Analysis results in 108.84 mm (stddev. 0.08 mm). Compared to the specification, which originates from functional requirements and tolerance synthesis the analyst would have accepted the assembly based on the CAT-results. Manufacturing and assembly simulation of the non-ideal parts shows that nearly all assemblies are out of specification (see Figure 9).

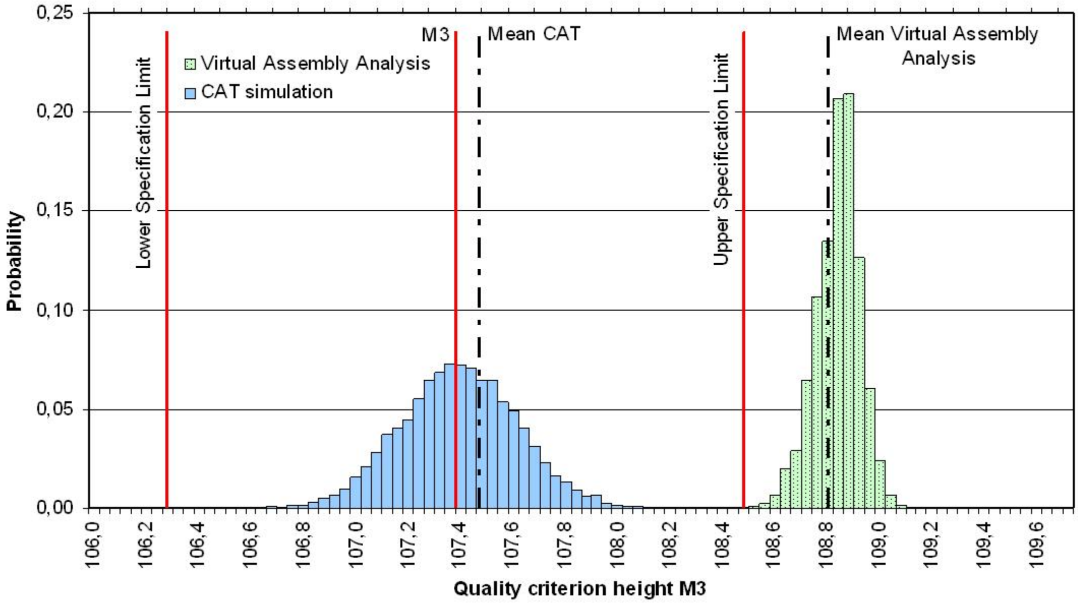

Figure 9. Histogram of height M3 obtained from CAT and Virtual Assembly Analysis

4. Summary and Outlook

In this paper, we presented an innovative method for the reliable and realistic assurance of geometric attributes of the designed product. Variational Finite Element Simulation is used to generate representative parts with manufacturing deviations. Latin Hypercube sampling is used to obtain a stochastic simulation response. The part variants are combined to non-ideal assemblies. A Relative Positioning algorithm assures the meaningful combination of the simulated parts without intersections, taking into account the wanted positioning scheme. The presented method greatly differs from the common computer-aided tolerancing methods: The analyzed geometry is not abstracted to ideal features. The input geometry for the virtual manufactured assembly analysis corresponds to more realistic deviation ranges due to FE-based manufacturing simulations. After the part positioning step, complete, non-ideal product variants can be visualised and measured concerning functional and aesthetic quality. The Virtual Assembly Analysis improves the selection of quality measures and gives a holistic impression of the resulting assembly deviations. Thus, the proposed method can be used to extend common CAT analysis, but can also be used independently. Because of the visual verifiability, it is less error-prone than common methods. It can easily be adopted if the company-internal virtual geometric assurance is based on FEA until now and thus successfully contributes to the delivery of high quality products and systems. The mentioned advantages justify the increased computational effort compared to CAT analysis. Next step is the experimental validation of the Virtual Assembly Analysis. Moreover, an FE-Simulation of the subsequent spot welding process is planned to address the effect of joining non-ideal parts. This will allow for the geometric analysis of the whole industrial manufacturing process from single part manufacturing to final assembly.

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