COMBINING STRUCTURAL COMPLEXITY
MANAGEMENT AND HYBRID DYNAMICAL
SYSTEM MODELLING

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1. Introduction
The engineering community faces new challenges due to a drastic increase in the scale and the
complexity of the processes and systems. Large, multidisciplinary and networked systems are affected
by reams of interactions within and between several domains. According to Strogatz [Strogatz 2001],
two of the meaningful kinds of complexity in such networks are structural complexity and dynamical
complexity. Already small variances can lead to an unwanted or unexpected structure or even to
undesired or unstable dynamics. Therefore, it is essential to get knowledge about both kinds of
complexity. Hence, modelling the system’s structure and its dynamics is gaining importance as it
improves the system’s understanding. Based on adequate models process parameter optimisation and
implementation of process improvement or innovation projects can be eased.

On the one hand, structural models – like the DSM (Design Structure Matrix) – have gained wide
acceptance throughout the product development and the industrial engineering communities
[Browning, 2001]. They allow complexity decomposition as well as a first qualitative analysis based
on structural system characteristics [Lindemann, 2009]. On the other hand, hybrid dynamical systems
are increasingly used in automation engineering. The dynamic of these systems can be generally
described by an interconnection of discrete and continuous dynamical subsystems. This allows for
representing a wide range of systems including industrial processes. Additionally, such a system’s
representation allows the application of a wide variety of dynamical analysis methods. Both kinds of
complexity are interrelated and should be analysed in a well-structured combination and not
sequentially like it is currently done.

However, both approaches are limited in their applications as structural complexity management
cannot describe the system’s behaviour and hybrid dynamical system modelling cannot be applied to
large scale or complex systems as it requires detailed and expensive to acquire data about the
interactions.

This paper presents a new approach, which combines structural and dynamical models in order to form
a unified system model. Thereby, both types of complexity can be analysed simultaneously. The paper
is structured as follows: First, a short review of the current research in structural and dynamical
modelling is presented. In section 3, major interfaces between structural and hybrid dynamical
modelling are shown and used to generate a new modelling process. Therefore, a new hybrid
modelling paradigm is introduced and combined with structural modelling. Finally, a simple example
considering only one dynamical domain is shown, in order to highlight the approach and thus the
potential of combining structural and dynamical modelling.
2. Structural and dynamical modelling

2.1 Structural modelling

Structural models are used to describe complex systems. Using matrix-based methods, systems can be broken down into smaller units, which are easier to handle. First, there is the Design Structure Matrix (DSM). It depicts the relations between system elements of one domain in a square matrix. Such domains are for example components of a product or engineers belonging to a design team. If a link exists between two elements, an entry is made in the respective cell of the matrix [Lindemann, 2009]. DSMs can be differentiated by their application. On the one hand, there are static matrices, for example containing the relations between components. They mostly serve product architecture, team-based or organizational purposes. On the other hand, DSMs can be time-based. For example, those are applied to problems concerning scheduling activities [Browning, 2001].

In order to model the relations between two domains, e.g. the members of a design team and the documents they produce, the Domain Mapping Matrix (DMM) can be used. In this matrix both domains are opposed and the links in-between form the entries in the matrix [Lindemann, 2009].

The third type, the Multiple-Domain Matrix (MDM) covers several domains. It is composed of DSMs and DMMs, which represent different views of a system in one model, e.g. components, people, documents and requirements. If one or two domains are linked by edges of different types, several subsets of data can be considered in a MDM. As it is presented in figure 1, for each edge type a separate DSM or DMM is created. Thus, the analysis of the MDM is conducted repeatedly in order to consider all possible combinations. Thus, this approach allows for decomposing, structuring and analysing complex systems [Lindemann, 2009].

![Figure 1. Example of a structural model – the multiple-domain matrix (adapted from [Lindemann 2009])](image)

The methods described above are related to graph theory. Edges and nodes of each matrix can be represented by a strength-based graph, which can be used to gain additional insights into the system by its visualisation [Lindemann 2009]. Figure 1 shows how different nodes of different domains and their edges are transformed into a MDM. According to [Lindemann 2009], several analysis approaches can be applied to these models. In summary, structural models built by DSMs, DMMs and MDMs can be applied not only to support decisions in product development concerning product structuring or organisational tasks in optimising the alignment of process steps, but also to get a thorough understanding of a system’s structure, its subunits and inherent dependencies.

When discussing the dynamics of a system, there are two major points of interest. First, there is the system’s evolution over time. Network theory addresses the evolution of the system structure. Network theorists developed a sophisticated toolset for predicting and simulating structural evolution [Cami 2008]. However, these approaches are often limited to graph models. In current research, efforts are made to enhance the DSM methodology in order to model the states of the same system at different points in time as DSMs, DMMs and MDMs. For example, [Eben 2008] describe an approach combining a ΔDSM – depicting edge changes – and a DMM – covering node changes – to show how a system grows or shrinks over time.
2.2 Dynamical modelling of complex systems

Structural models have the ability to show dynamical regions but up to now only in a static manner. Those regions are in the literature often highlighted to be modelled and simulated using well-known tools like System Dynamics. However, those tools require a very deep knowledge of the process interactions, are fixed in one dynamical domain and the question of analysis-oriented transformation from the structural toward simulation model is still in its infancy. Additionally, exclusively trusting simulations can result in drawing false cause-and-effect-conclusions, a risk that grows exponentially with the system’s complexity. Moreover, nonlinear dynamical phenomena, like bifurcations or chaos [Wiggins, 2003], are hard and/or time-consuming to be fully understood by only using global simulations. The bifurcation theory deals with the parametric stability behaviour of nonlinear dynamical systems, which is of particular importance for adapting, improving or innovating multi-parametric processes. In Figure 2a the core of the classical V-model adapted from [Norwig, 2002] is shown. It clarifies the modelling process of complex dynamical systems, which are primary used for simulations. It can be seen, that the feedback to the structural design is only based on the simulation results. Besides the already mentioned disadvantages considering only this link, also a long period of time is passing between structural changing iterations.

Figure 2. Modelling complex dynamical systems

Figure 2b shows the general characteristic of combined discrete and continuous dynamics according to a hybrid dynamical system’s model. Although hybrid dynamical systems are gaining importance in lots of areas and offer the potential to tackle all of the above mentioned problems, there are several open problems regarding their systematical design and their analysis opportunities. Currently, there exists no common definition for this type of systems. Hence, various different modelling paradigms have been proposed (see [Buss, 2002] and the references therein). Well-known hybrid modelling paradigms combine two basic mathematical system representations e.g. ordinary differential equations and discrete events (DEVS) or petri nets. However, for tackling a wide range of complex systems this strategy needs to be enlarged. Accordingly, an earlier feedback for adjusting the structural design in consequence of the system’s performance is required. Hence, the integration of sub-models has to be already feasible at mathematical modelling considering a projection onto structural models. Additionally, such a linking enables an automated adjusting of the reachable pattern quality based on structural (qualitative) dynamical criteria.

3. Combining structural and hybrid dynamical system models

As mentioned, a well-structured combination of qualitative and quantitative modelling is gaining importance, with the aim of keeping the balance between simulations and theoretical analyses. This new challenge for systems research requires new computer-based technologies for assistance and partial automation of the modelling process. A novel approach of combining is introduced in this section.

3.1 Interfaces between structural and hybrid dynamical system models

A unified modelling method allowing a mathematical representation of a wide range of complex systems or processes needs to include all of the possible dynamical interactions in one hybrid system.
paradigm. Hence, this description form, denoted as multi-hybrid system, enables to subdivide the system into dynamical domains according to the available knowledge and data. This leads to three dynamical domains (subsystems): The discrete subsystem, which allows the description of discrete-time variable characteristics between system items, the continuous subsystem allowing continuous-time variable characteristics and the finite automation. The discrete subsystem is thereby given by a classical discrete-time dynamical expression and analogous the continuous subsystem by ordinary differential equations. The finite automation allows a system’s theoretical description of system’s parts requiring a minimum of knowledge e.g. using markov chains or petri nets. Such a multi-hybrid system allows for handling almost any kind of system information (e.g. functional, proportional, expert and linguistic knowledge), integrating them in a mathematical model and thus use them for the dynamical analysis. Figure 3b visualises a first approach of a multi-hybrid system paradigm given by:

\[
\begin{align*}
\dot{x} &= f(x, u, q, \theta, t), \quad q^{k+1} = g(x, v, q, \theta)^k, \quad \theta = s(n, w, x, q, t) \\
[\begin{pmatrix} x(t^+) \\ q(t^+) \end{pmatrix}] &= \Phi(x, u, q, n, t), \\
[\begin{pmatrix} n(k^+) \\ q(k^+) \end{pmatrix}] &= \Psi(x, u, q, n, t), \\
[\begin{pmatrix} x(t^+) \\ n(t^+) \end{pmatrix}] &= \Gamma(x, u, q, n, t) \\
y &= h(x, u, q, v, n, w, t)
\end{align*}
\]

Equation (1a) describes the three possible dynamical subdomains, equation (1b) the mapping functions (or jumping maps) between them (see [Buss 2002]) and equation (1c) the system’s output. Additionally, if the mapping functions are only activated by special events, activation functions can be added as restrictions.

Figure 3. Multi-Dynamic Mapping (MDM/Multi-Hybrid System)

Clearly, the potential of a theoretical analysis as well as for simulations deepens highly on the available level of detail of the system’s interactions. Additionally, systematic complexity decomposition and analysis-orientated division of the overall system into the dynamical domains of the multi-hybrid system are required. Figure 3a shows the layout of the MDM in order to support such a multi-dynamical mapping. The geometric shapes represent the different dynamical domains, which correspond to the shapes in Figure 3b. Each domain is modularly modelled in a separate DSM on the main-diagonal and their interactions are placed in the corresponding off-diagonal cells. Accordingly, a combination of structural and system’s theoretical modelling is achieved, allowing a direct transferability of their analysis. Based on such an integration of structural and dynamical behaviour, the system understanding and thus parameter optimisation and process improvement or innovation projects can be eased, while avoiding several iteration loops. In figure 4 the modified V-model of figure 2 is depicted, which clarifies the direct interaction between structural and system’s theoretical modelling for tackling the problems according to section 2.2.

In [Diepold 2009a] a structured process modelling approach by extending the DSM to the so-called DynS-DSM (Dynamical System Design Structure Matrix) is introduced. By keeping the properties of the DSM during the whole modelling procedure the approach allows a direct transferability of structural analysis or structural changes onto the system’s dynamics. Additionally, it supports the
parameterisation and the implementation process (figure 4) by integrating qualitative and quantitative system’s knowledge and allowing a system’s order reduction. The approach results in a discrete-time dynamical representation of the system’s significant dynamics. The potential according to theoretical analysis is also shown based on an illustration example considering bifurcations. As this approach is still fixed in the discrete-time domain a further enlargement needs to be done. However, this approach shows the potential of combining structural and mathematical modelling and can be directly integrated as part of the multi-hybrid system of figure 3b.

Figure 4. Balanced modelling approach for complex dynamical systems

In [Diepold, 2009b] a framework for a DSM-based pre-modelling analysis of complex systems is shown. It estimates the reachable pattern quality in order to get an early knowledge about the model reliability. Hence, a further data collection can be done if required in order to improve the aspired model before the modelling has even begun and thus allowing a drastic time-saving. The acquisition of data can further be focused on areas, which have a high impact on the model quality. Thus, the framework acts as a link between structural analyses and mathematical modelling, by supporting the detailed design and the specification process (see Figure 4).

Figure 5. Modelling process for multi-dynamic mapping

3.2 Transformation process from structural toward hybrid dynamical system models

As discussed above, the major interface between structural and hybrid dynamical system models are structurally significant parts of the systems. After determining those parts by structural analysis models they are transformed into a dynamical system representation. Figure 5 shows the resulting process. It is based on the standard process of structural complexity management as proposed in [Lindemann, 2009]. However, the new process does not contain all activities proposed in [Lindemann, 2009]. The activities deduction of indirect dependencies and product design application are not shown in figure 5. Deduction of indirect dependencies is viewed as part of data acquisition and product design application is replaced by the modelling, simulation and interpretation activities. The activities of modelling and simulation are discussed in more detail in the sections 2.2 and 3.1. The interpretation
activity transforms the results of the simulation and the findings from the mathematical analysis, in particular the determined system behaviour, to answer to the initial problem to be solved. For example, new approaches for dealing with the system can be derived from the behaviour. Significant structures in the context are those, which are relevant for the system’s dynamics and behaviour. One criterion which is particularly important for the dynamics is the cycle criterion as it is fundamental for feedback loops. [Diepold, 2009a] use cycles to derive the subsystems, which are significant for the overall system behaviour. However, as there are numerous structural criteria [Lindemann, 2009], which can be relevant for the system’s dynamics, choosing the right criterion for the specific purpose is a critical decision during the structural analysis. In most cases the cycle criterion has to be considered.

4. Structural and dynamical analysis of a ball-pen
In this section, the introduced combination of structural and dynamical analysis is applied to a simple example in order to comprehendingly show its potential. First, the example product itself, a ball-pen, is introduced. Thereafter, the structural model from a component and a parameter point of view is analysed. Based on those results, a structurally significant subsystem of the ball-pen is detected and modelled as a dynamical system by adjusting the pattern.

4.1 The ball-pen
The ball-pen consists of seven components. The head piece and casing function as the outer shell of the pen, which hold all other components. The spring is brought onto the lead, in order to push back the latter as long as it is to be stored inside the shell. Both are inserted into the head piece. Thus the head piece is in physical contact with the spring, and the spring is in contact with the lead. The casing holds the button, which is inserted into the latter. The button serves to move the tip of the lead outside the head piece and hold it in this position against the spring’s force. When head piece and casing are assembled, the ring is positioned between these components. Each of the three items is in contact with the other two. Finally the clip is attached to the casing. The components’ physical contact is not only a static dependency but also a dynamical one. One instance is the length of the lead, which is of major influence to the length of head piece and casing. Further a change of the inner diameter of the latter has an impact on the head piece’s outer thread diameter as well as on the inner diameter of the ring.

4.2 Structural analysis
Figure 6 shows the component DSM of the ball-pen. A cross in the matrix indicates the two adjacent components are in contact. The DSM is symmetric, so all links are bidirectional. This limits the applicable structural criteria. In symmetric matrices mainly clusters and cycles are applied. Other criteria such as paths and degrees of nodes play a far lesser role.

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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>Clip</td>
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Figure 6. Component DSM of the ball-pen

Thereby, the original component DSM is transformed to a parameter DSM. Moreover, the relationship type modelled in the matrix is changed. The original matrix represents contact relations whereas the parameter matrix represents functional dependencies. The resulting DSM is partly shown in Figure 8a (see [Diepold, 2009b] for the complete matrix).

Besides the differences in the content and interpretation, the parameter DSM differs structurally fundamentally from the component DSM: The component structure is connected and more densely linked, the parameter DSM consists of eight independent structural components. The differences can
be explained by the structure of the dependencies of single components parameters. They are mostly independent, which means they are not connected. Therefore, the parameter structure is rather sparse. Table 1 shows the analysis results for both structures for a range of structural criteria.

Table 1. Results of the analysis of the ball-pen’s structure

<table>
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<tr>
<th>Structural Criterion</th>
<th>Component Structure</th>
<th>Parameter Structure</th>
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<tbody>
<tr>
<td>Number of Elements</td>
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<td>27</td>
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<tr>
<td>Number of Relations</td>
<td>20</td>
<td>44</td>
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<tr>
<td>Number of Components</td>
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<td>8</td>
</tr>
<tr>
<td>Number of Cliques</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Number of Cycles</td>
<td>32</td>
<td>36</td>
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</table>

The structural analysis reveals the most influential parts of the structure. Especially, the distribution of the cycles among the dependencies of the parameter structure allows for deriving a subset of elements and relations, which is particularly important for the dynamical behaviour of the ball-pen. The complete procedure of deriving this subsystem is described in [Diepold, 2009a]. In case of the ball-pen the procedure results in a subsystem of ten elements, which are shown in figure 8a. The crosses are replaced by the corresponding functional dependencies. This subsystem is further considered for dynamical modelling.

4.3 Dynamical analysis

The dynamical analysis is done for two clusters (A, B) and for simplicity, only the discrete-time domain of the multi-hybrid system according to figure 3 is considered. Cluster A containing only two elements is a virtual one only used to clarify the possible negative effects of an undirected interaction in combination with a linear interaction approximation. Based on cluster B (see figure 8a), which equates to the structurally importuned cluster considering cycles (see section 4.2), the effects according to cluster A are studied and reflected to adjusting the pattern quality.

The geometric interactions are modelled considering two different levels of detail: A proportional approximation and an adequate functional approximation of the exact relation. The proportional approximation is chosen to

$$\forall b < a \Rightarrow b = 0.5 \cdot a, \; \forall b > a \Rightarrow b = 2 \cdot a,$$

where a and b represents the corresponding elements of the considered DSM. The considered exact functional relations as well as their approximations are depicted in figure 7.

Hence, the element b changes not until the element a reaches a critical value $\lambda_c$. The functional approximation is considered for simulation here, because the exact functional interaction is a $C^0$-function and thereby disadvantageous for a further mathematical analysis, which is not included in this paper. For the functional approximation $b > a$ a fourth order polynomial and for $b < a$ an exponential function were chosen:

$$b \approx -2.60e^{-3} \cdot a^3 + 4.49e^{-4} \cdot a^2 + 1.08 \cdot a + 0.21$$

$$b \approx 13 \cdot \left(1 - e^{-\frac{a}{9.1}}\right)$$

Figure 7. Functional approximation for mathematical modelling
The stability behaviour of an autonomous linear system is divisible in three characteristic behaviours: Either it reaches zero as an asymptotic stable fixed point or it stays in a periodic orbit or it becomes unstable. This is clarified by cluster $A$. Formula (4) shows three linear system’s matrices $i$ of this cluster according the dynamical behaviour $x^{k+1} = A_i^k \cdot x$. The state variables $x$ can be directly interpreted as DSM-elements. The variable $k$ stands thereby for the iteration parameter of the discrete-time mapping (see [Wiggins, 2003]).

In $A_i^k$ the interactions according to equation (2) are used and slightly modified for $A_{2}^k$ and $A_{3}^k$.

$$A_1^k = \begin{bmatrix} 0 & 2^k \\ 0.5 & 0 \end{bmatrix}, \quad A_2^k = \begin{bmatrix} 0 & 2.1^k \\ 0.5 & 0 \end{bmatrix}, \quad A_3^k = \begin{bmatrix} 0 & 1.9^k \\ 0.5 & 0 \end{bmatrix}$$

Formula (5) shows the corresponding pairwise eigenvalues and the stability conclusion.

$$\xi_i = \pm 1 \text{ (limit cycle)}, \quad \xi_i = \pm 1.02 \text{ (unstable)}, \quad \xi_i = \pm 0.98 \text{ (stable)}$$

Hence, a simple linear approximation of an engineered networked system requires suitable proportional coefficients. Additionally, it allocates no stable and unstable regions. Accordingly, the question of how to check the pattern quality arises for complex engineered systems.

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Figure 8. Dynamic behaviour of the ball-pen’s core components

Figure 8a shows the significant cluster $B$ of the ball-pen’s DSM, which was detected using the framework [Diepold, 2009b] (see section 4.2). The determined level of detail of each interaction, considering the two already mentioned approximation levels, is done based on the amount of cycles the corresponding interaction is involved in. This is also directly transferred from the framework, where finally 0.5 represents proportional and 1.0 functional interaction knowledge (see figure 8b). By applying the introduced relations (see equations (2) and (3)) and modelling in the discrete-time domain a mathematical representation is achieved, which is adjusted to structural criteria. For instance, the dependence of parameter 7 on parameter 2 should be modelled functionally.

Based on that, Figure 9a shows the simulation results for parameter 7 of Figure 8 considering various levels of interaction knowledge. Therefore, the qualitatively determined knowledge according to figure 8b, is considered as 0% of error. The more functional interactions are transformed into proportional ones, the higher the qualitative modelling error is. Figure 9b shows the detected error areas, where the modelled system stays stable, turns into a periodic orbit or becomes unstable. The dashed-dotted line in figure 9a clarifies the continuous growing of the unstable simulation. As both, periodic orbit or unstable is clearly not the desired system’s performance of the ball-pen, adjusting the pattern quality to structural analysis results in a suitable pattern quality or model validation.

### 5. Discussion

The example highlights the process shown in figure 5. By applying several structural analyses to the ball-pen’s parameter structure, the most critical areas of the structure are determined. These areas
comprise most of the structural criteria, which dominate the ball-pen’s dynamics. In this case the dynamics are dominated by cycles which are mostly situated in a substructure of ten elements. Thus, the structural analysis allows for qualitatively determining those areas which are relevant for a more in-depth analysis of the system dynamics. Additionally, the mathematical pattern quality should be aligned to structural-based qualitative error estimation (figure 9b). Thereby, the overall costs for modelling the system’s dynamics are reduced, as the costs for structural modelling are lower than those for a detailed mathematical modelling.

The mathematical modelling process includes new data into the chosen subset of the structural model. For example, it includes assumptions or existing knowledge about the dynamical behaviour of the system elements. The mathematical model of the ball-pen describes the changes of the parameters as a reaction to changes of connected parameters. The analysis of the extended model allows for making predictions concerning the stability of the ball-pen when facing changes of the environment. Moreover, the model itself can reveal the necessity for introducing or acquiring more data. If the predictions reveal for example that, the system is close to a bifurcation point more data is necessary to decide which path will be taken. The results of the mathematical analyses can be fed back to the structural design of the system. As the mathematical models are structural equivalent to the structural model according to figure 3, they allow for proposing sensible or required changes to the system structure. Thereby, undesired behaviour of a system can be traced back to the underlying structure, which then can be adapted to suit the dynamical requirements.

6. Conclusion

Structural complexity management and hybrid dynamical system modelling were concerned with in this paper. Structural complexity management focuses on qualitatively describing the system elements and analysing their connectivity, whereas hybrid dynamical system modelling puts the system’s dynamic, stability and robustness in focus. However, both approaches are limited as structural complexity management cannot describe the system behaviour and hybrid dynamical system modelling cannot be applied to large scale complex systems as it requires detailed and expensive to acquire data about the interactions.

A process to combine both methods to a unique model for tackling complex system’s modelling and analysis tasks has been introduced. Different dynamical domains have been integrated in order to embrace the high different of used information in a large multidisciplinary network. The direct connection of the resulting multi hybrid mathematical model to the MDM has been shown by sub-structuring the system. This allows an earlier feedback for adjusting the structural design in consequence of the system’s performance and enables the use of structural analysis as a pre-modelling. This has been shown by applying the approach to a simple example considering only one dynamical domain. Thereby, the structural analysis has been used to detect important subsystems and
transform the results into a moderate mathematical model. The potential of this procedure as a qualitative analysis of the pattern quality was shown by simulation results. More detailed case-studies will be done in future work to test and optimise the approach in several areas of interest considering real hybrid systems. The investigated systems are situated e.g. in products design, manufacturing systems, systems engineering, requirements engineering and social networks. The case-studies will be part of the Collaborative Research Centre SFB 768, which has been introduced in section I.

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