# SIMULTANEOUS CONSIDERATION OF PRODUCT DESIGN AND END-OF-LIFE RECOVERY NETWORK DESIGN

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### ABSTRACT

In recent years, product recovery has become a great concern to manufacturers who take the responsibility for product end-of-life decisions. Product design and reverse logistics network design are the major determinants of recovery profit, and each is related to the other. Thus, in order to evaluate the recovery profit and eventually maximize it, an integrated approach must be developed so that both product and network design are considered concurrently at the product design stage. In this paper, a generic mathematical model for optimizing recovery network design is proposed that reflects the impact of product design during network optimization by using a transition matrix. It provides not only the optimal network design but also the optimal recovery plan for a large number of products as well as the expected recovery profit of that plan. The proposed model is expected to provide the optimal solution that corresponds well to a real problem. For the purpose of validation and verification, the proposed approach was applied to a simulated cellular phone recovery case.

Keywords: product recovery, reverse logistics network design, product end-of-life decision making

## **1** INTRODUCTION

A product that has exhausted its physical and/or functional lifespan is supposed to reach its retirement stage. As environmental regulations urge stronger stewardship for product retirement, disposal can no longer be the primary retirement strategy, and companies need to find a way to reduce waste and save resources. Recovering used products is a promising solution to this problem. Various recovery options, including reuse, repair, recycling, and remanufacturing, enable companies to comply with legislation while gaining some economic advantage as well. At comparatively little cost, companies can utilize the useful resources remaining in used products. In this respect, product recovery has become a field of rapidly growing interest for product manufacturers. Instead of disposal, more companies have been choosing recovery as their primary retirement strategy [1].

Environmental regulations are strong negative motivators for companies to undertake product recovery. However, in order to facilitate product recovery and sustain its growth, companies should be positively motivated so they will pursue recovery voluntarily. Thus, as companies seek economic incentives, engineering methods for maximizing recovery profit are in great demand.

Product recovery is the process of collecting used products from their former users, sending recoverable units to recovery facilities, reprocessing collected units to render them re-marketable, and distributing recovered products, components, or materials to future customers [2]. To maximize the profit of this process, companies should make the cost of transportation and reprocessing minimized, while the revenue from selling the recovered units is maximized. What is most important in achieving this goal is product design [3-5]. Product design features, including product architecture, functional performance, and material properties, greatly affect what the possible recovery options are for a product and how profitable they can be. Therefore, companies need to identify optimal product design that possesses maximum recovery potential.

With this aim in mind, a large number of studies have been conducted in the areas of product design. They have considered product design improvement as a way to enhance recovery profit and have sought optimal or at least better product design in pursuit of less recovery cost and/or more recovery revenue. Design for disassembly (DfD) is a representative design method in this area [6, 7]. The primary purpose of such methods is to evaluate a given product design in terms of ease of recovery and to suggest

redesign directions based on ease of recovery. Various approaches have treated DfD with an assumption of a fixed recovery plan [8-10]: they focused on evaluating the disassembly time and cost of a product in order to find design weaknesses that counter the fixed recovery plan. Some approaches have proposed modular design, where similar components sharing common characteristics with respect to an intended recovery plan are clustered into a module [11, 12]. Applying these methods iteratively while changing the input recovery plan, designers can find the optimal (or at least a better) product design that leads to higher recovery profit. Some researchers have developed methods that find the optimal recovery plan first, and then suggest desirable design improvement, so the designers know the maximum recovery potential of the current product design, including a detailed reprocessing plan and expected profit [5, 13-15]. With this information, they can improve the current design rather incrementally by means of, for example, modular design, assembly method changes, material changes, configuration changes, and so on. Previous methods evaluate design alternatives based on maximum recovery profit. Although several studies have demonstrated ways to evaluate recovery profit by finding the optimal recovery plan, they have limitations in that they have overlooked the impact of recovery network design. Recovery profit consists of the cost of transportation and reprocessing as well as the revenue from selling the recovered units. Thus, the recovery profit of a product is affected by both product design and recovery network design; they are tightly coupled problems. Specifically, a change in recovery logistics network features has impacts on the product design optimization since it changes the expected recovery profit. For example, changes in such network features as facility locations, facility capacities, and feasible operations at facilities can alter reprocessing costs, transportation costs, or recovery revenue of a product. The reverse is also true, as a change in product design affects the optimal network design and its recovery profit. For instance, changes in product design features, such as architecture, material property, specifications, and others, imply the shift of possible recovery options and their necessary recovery operations. This shift induces different transportation and reprocessing costs as well as different recovery revenue from a network design.

Despite the fact that product design and network design are interwoven, previous methods dealt with product design and recovery network design separately. They assign the logistics costs before finding the optimal recovery plan. If there is a predefined fixed network design, it might be reasonable. However, it seems more realistic that the logistics cost changes according to changes in recovery plan. Also, network features make the current methods have limitation in that they only consider a single product. In reality, even same products can be recovered differently with various plans due to facility capacity or market demand. However, previous methods give only one plan for same product since they ignore such network features.

A similar problem exists in another major area of recovery research – the recovery network design field. Here, the goal is to find the optimal reverse logistics network design, that is, the optimal path to distribute returned products so as to meet the demand for recovered products. Major decision variables are the volumes of product flows between two facilities, and the key constraints are the capacity of facilities, i.e., maximum amount of product that can be handled by facilities. General network design models [16-18] neglect product design when optimizing a network. All the process in which a product passes through in its recovery is modeled as an intermediate node, as if it is an operation in a plant. Final output of recovery is identical regardless of node. This makes it impossible to consider how design difference of products affects the network design. Although a few studies have suggested a model that incorporated product features [19, 20], however, most of them are case-based and, for that reason, they lack generality.

In order to evaluate the recovery profit and eventually maximize it, product design and network design should be considered at the same time. In other words, recovery network design should be considered at the design stage as a set of decision variables that would be optimized with the recovery plan. In this work, an integrated approach is developed which considers both product design and network design simultaneously. A generic mathematical model for optimizing recovery network design is proposed that reflects the impact of product design during network optimization by using a transition matrix. It gives not only the optimal network design but also the optimal recovery plan for a large numbers of products and expected recovery profit of that plan. This method contributes to better design solutions to maximize product recoverability. For a given product design, by applying this method for different designs, company can obtain design alternative set, which consist of a product design and accompanied optimal network design. Each alternative also shows its expected recovery profit. Based on this information, company can select best design set that has the highest recovery potential.

The rest of the paper is organized as follows. Section 2 gives an overview of product recovery logistics network followed by the transition matrix -a key design enabler -in Section 3. A mathematical model is presented as a design optimization problem in Section 4. An illustrative case is presented in Section 5 followed by conclusion.

# 2 PRODUCT RECOVERY LOGISTICS NETWORK

A product recovery system considered in this research is represented in Figure 1. Usually, five types of facilities are involved in product recovery: collection centers, disposal sites, warehouses, recovery plants and demand sites. Collection centers are a central point where used products are first collected from customers at different locations. After tests (i.e., assessment of product's status) at this collection point, recoverable products are transferred to a recovery plant. For later recovery, some of them can be stored at a warehouse. Unrecoverable units move to disposal sites for landfill or incineration. Recovery plants reprocess returned units and change them into re-marketable units. Sometimes more than one recovery process at different recovery plants are required to change a unit into a desirable form. When component recovery is more worthwhile than product recovery, disassembly is performed in advance of other recovery processes. In disassembly, an item is dismantled and turned into a set of "child" subassemblies. This disassembly operation is mostly affected by the way product is designed. After disassembly, individual child subassemblies continue their recovery as independent units.

There are various reprocessing options, including reuse, repair, recycling, and remanufacturing [21]:

- Reuse: An item is used for its original purpose without the need for repair operations. If necessary, it may go through some cleaning process and/or minor maintenance.
- Repair: An item is restored to be a working unit in terms of its original functions.
- Recycling: An item undergoes shredding and/or separation treatment to recover raw materials. Incineration to produce heat and electricity is also included in this option.
- Remanufacturing: An item maintains its identity and structure and is refurbished or upgraded as a new product. Disassembly, overhaul, and replacement are part of remanufacturing a product.

A recoverable product changes to a set of serviceable units after all reprocessing. Finally, recovered units are sent to various demand sites, such as manufacturing plants and remanufacturing markets to be sold.



Figure 1. General structure of product recovery (revised from Fleischmann et al. 1997)

# 3 TRANSITION MATRIX FOR MATHEMATICAL MODELING

Product recovery is much dependent upon the way a product is designed. Product design decides what kinds of recovered products can be produced and what recovery operations are necessary to produce them. In the product recovery, network design is the factor deciding the feasibility of recovery operations as well as the profitability of possible recovery plans. Network features make the recovery cost and revenue differ; what facilities are involved in the network, what sorts of and how well the facilities perform recovery operations, which facility is assigned to do a particular job, what customers are included as end nodes of network, and etc. All these affect operation cost, transportation cost, and/or recovery revenue.

To capture this characteristic, an optimization model based on a transition matrix is presented in this paper. Transition matrix is a matrix that represents every possible recovery scenario a product design derives. Specifically, a transition matrix enables a mathematical modeling of the relationship between product design and recovery processes.

In previous research [5, 22], a transition matrix has been used to model product disassembly. It represents the AND/OR graph of a product in the form of matrix. As for a given product design, all

feasible subassemblies and feasible disassembly transitions are enumerated so that they constitute rows and columns, respectively. The cell values are assigned to represent how a transition changes a parent subassembly into a set of child subassemblies. Accordingly, the transition matrix contains information about every possible disassembly scenario that can be expected from a product design.

In this research, a transition matrix is modified so its transitions are defined for disassembly operations as well as for other reprocessing operations. A transition, which means a recovery operation, changes a product's state into another type. For example, an operation for remanufacturing changes an old item into a new item, and an operation for disassembly transforms an item into a set of its subassemblies. Since the feasibility of a transition and every possible state that can result from transitions are affected by product design, different designs result in different transition matrices.

Table 1 shows the transition matrix of a returned product. Every possible state a returned product can take on the recovery network is defined as a state, s. The whole set of feasible states constitutes the rows of the matrix. The columns show feasible transitions, namely, recovery operations, p. In the example are five possible operations. Each cell in the matrix has an integer value. Values in a column describe the input and output of the corresponding operation. If a cell (s, p) has a value of -1, a unit having state s is processed according to operation p. Alternatively, if a cell (s, p) has a value of 1, a unit having state s is generated also according to operation p. If a cell (s, p) has a value of 0, represented as a dot in the table, such state has nothing to do with the operation p. In short, the transition matrix shows which operation is needed to transform a parent unit in a certain state into a certain set of child units in other states. For example, in Table 1, a unit in state 1 is transformed to a unit in state 2 by means of recovery operation 1; a repair operation changing a failed unit to a functioning unit might be represented in this way. Operation 2 changes a unit in state 1 into two units in states 3 and s; for example, a disassembly operation can be represented in this manner. Operation p converts two units in state 3 and s into a unit in state 1; a reassembly operation that occurs in remanufacturing can be represented like this.

	<b>Operation1</b>	<b>Operation2</b>	 Operation p
State 1	-1	-1	1
State 2	1		
State 3		1	-1
State s		1	-1

Table 1. Transition matrix for a returned product

	Operation1	Operation2	 Operation <i>p</i>
$u_{jp}$	0	500	500
C <sub>jp</sub>	$\infty +$	10	25

Table 2. Capacity and capability of a recovery plant

Table 2 shows the important network features, capacity and capability of recovery facilities, which are defined for each of facilities and each of transitions. Each recovery plant has different capabilities as well as different capacities, and both of them are the major features in a logistics network. Capacity  $u_{jp}$  for plant *j* operation *p* indicates the maximum amount a facility can handle at one time. In contrast, capability indicates whether a facility has the ability to do the job, and if so, how well. Although previous research has focused only on capacity information, capability is also very important in practice. There could be a special recovery operation only a few factories can perform; even when both facility A and facility B can perform the same recovery operations, the cost could be different. In the proposed model, capability is reflected through the unit operation cost  $c_{jp}$ . High capability is reflected through a low operation cost, and vice versa. If a facility cannot perform an operation 1 in Table 2.

The connection between the transition matrix and the facilities' capacity and capability information is formulated by equations (1), (2), and (3), where  $z_{jp}$  is a decision variable indicating the number of times operation p is executed at recovery plant j. Equation (1) shows the balance between inflow  $E_{js}$  and outflow  $O_{js}$  of product in state s at facility j.  $E_{js}$  and  $O_{js}$  represent total volume of input and output units in state s at the facility respectively under the transition matrix entry  $T_{sp}$ . Equation (2) shows the capacity constraints, and equation (3) shows the total operation cost of a network. Details of the symbols are described in the next section with the mathematical model.

$$E_{js} + \sum_{p=1}^{N_o} z_{jp} \cdot T_{sp} = O_{js} \qquad \forall j \in J, \forall s \in S$$

$$\tag{1}$$

$$z_{jp} \le u_{jp} \qquad \forall j \in J, \forall p \in P \tag{2}$$

$$C_{operation} = \sum_{j=1}^{N_f} \sum_{p=1}^{N_o} c_{jp} \cdot z_{jp}$$
(3)

# 4 MATHEMATICAL MODEL

### 4.1 Problem statement

The proposed method for simultaneously considering product end-of-life design and recovery network design is summarized as the following optimization problem:

### Given

- Transition matrix and the amount of returned products.
- Location and distance information of the potential recovery facilities.
- Cost of facility opening, recovery operations, and transportation; revenue from recovered items.

### Find

- Facilities to be opened or used and the volume of items flowing from one facility to another.
- Recovery operations performed by each facility and their frequency.

### Subject to

- Flow balance feasibility: an item must be sent only to an available facility that is open or in use; also, a facility should maintain its flow balance between input and output units.
- Facility capacity: relative to a recovery operation, a plant has its own capacity, and it can deal with only the amount less than its capacity. A plant has zero capacity for its unavailable operations.
- Unit state change feasibility: a recovery operation converts a single item into other unit(s). This state change should be feasible.
- Avoiding excess fulfillment: The supply of a recovered unit cannot exceed the demand for the unit.

### Maximizing

• Recovery profit expected from an amount of product with a given design.

### 4.2 Nomenclature

### Index Sets

$I = \{1, \cdots, N_c\}$	collection points; index $i \in I$
$J = \{1, \cdots, N_f\}$	potential locations of recovery plant; index $j \in J$
$K = \{1, \cdots, N_d\}$	fixed demand locations; index $k \in K$
$L = \{1, \cdots, N_g\}$	potential locations of disposal site; index $l \in L$
$R = \{1, \cdots, N_w\}$	potential locations of warehouse; index $r \in R$
$P = \{1, \cdots, N_o\}$	possible recovery operation; index $p \in P$
$S = \{1, \cdots, N_a\}$	possible states of product on the recovery network; index $s \in S$

### Variables

$X_{ils}^{g}$	volume of product in state <i>s</i> flowing from <i>i</i> to disposal location <i>l</i>
$X^w_{irs}$	volume of product in state s flowing from i to warehouse r
$X^{\alpha}_{ijs}$	volume of product in state <i>s</i> flowing from <i>i</i> to recovery plant <i>j</i>
$X_{iks}^{\beta}$	volume of product in state <i>s</i> flowing from <i>i</i> to demand site <i>k</i> directly
$X_{jks}^{\gamma}$	volume of unit in state <i>s</i> flowing from plant <i>j</i> to demand site <i>k</i>
$X_{i_m i_n s}^{\delta}$	volume of unit in state <i>s</i> flowing from $j_m$ to $j_n, j_m \neq j_n$
$X_{js}^{h}$	volume of unit in state s not proceeded anymore and discarded at j
$Y_{j}^{f}, Y_{l}^{g}, Y_{r}^{w}$	indicator opening recovery plant <i>j</i> , disposal location <i>l</i> , and warehouse <i>r</i> respectively
$Z_{jp}$	number of times operation $p$ is executed at recovery plant $j$

### Parameters

$T_{sp}$	entity value of transition matrix
$E_{is}$	total volume of returned product with state s at collection point i
$\mathcal{V}^{d}_{ks}$	volume of demand for unit in state <i>s</i> at site <i>k</i>
$u_{jp}^{f}$	maximum capacity of recovery plant <i>j</i> for recovery operation <i>p</i>
$r_{ks}^d$	revenue from providing a unit in state s at demand site k
$c_j^f$	fixed cost for opening recovery plant j
$\mathcal{C}_l^{g_1}, \mathcal{C}_r^{w_1}$	fixed cost for opening disposal location l and warehouse r
$C_{il}^{g_2}, C_{ir}^{w_2}$	unit transportation rate from collection point $i$ to disposal location $l$ and warehouse $r$
$c_l^{g_3}, c_r^{w_3}$	unit processing cost at disposal location l and warehouse r
$c^{\alpha}_{ijs}, c^{\beta}_{iks}$	unit transportation rate from collection point <i>i</i> to recovery plant <i>j</i> and demand site <i>k</i>
$c_{jks}^{\gamma}$	unit transportation rate from recovery plant $j$ to the demand site $k$
$c^{\delta}_{i_m i_n s}$	unit transportation rate between recovery plants from $j_m$ to $j_n, j_m \neq j_n$
$c_{jp}^{o}$	unit processing cost for recovery operation p at recovery plant j
$C_{js}^{h}$	unit penalty cost for the unit with state s not proceeded at plant j

### 4.3 Objective function

The objective of this model is to maximize the profit from product recovery. Conversely, it is to minimize the total recovery cost after deduction of the total revenue (*R*). In this model, total recovery cost is the sum of eight cost components (Detailed descriptions are given below.): cost for site opening ( $C_1$ ), cost for disposal ( $C_2$ ), cost for storage ( $C_3$ ), cost for transportation ( $C_4$ ,  $C_5$ ,  $C_6$ ), cost for recovery operation ( $C_7$ ), and penalty cost ( $C_8$ ) for unprocessed or discarded products. The objective function is modeled as shown in Equation (4).

$$\min f: \sum_{n=1}^{8} C_n - R \tag{4}$$

#### Site opening

A returned product reaching the collection point i is sent to another place in order for further recovery processes. There are three different types of site where the used product can be transported to; recovery plant, disposal site, and warehouse. What should be considered here is that a product can be transferred only to an available place. Perhaps, a site is constructed by the company. Or, a site can be used by the company under some contracts with the site owner. In such cases, company should pay some fixed costs. Equation (5) represents this fixed cost, where Y is a binary variable indicating whether a site opens or not.

$$C_1 = c_1^f Y_1^f + c_l^{g_1} Y_l^g + c_r^{w_1} Y_r^w$$
(5)

### Disposal from collection sites

A returned product can be thrown away at a disposal sites after it is tested/inspected at a collection sites. The disposal cost consists of transportation cost and processing cost. The former is for moving a product from collection point to disposal site, and the latter is for doing actual jobs for disposal, such as landfill or incineration. Equation (6) represents disposal cost.

$$C_2 = \sum_{i=1}^{N_c} \sum_{l=1}^{N_g} \sum_{s=1}^{N_g} (c_{il}^{g_2} + c_l^{g_3}) X_{ils}^g$$
(6)

#### Storage

Instead of throwing a product into the recovery network, company can suspend the decision and store the product for a while for some reasons. In this case, company should pay for the storage cost composed of transportation cost and warehousing cost. Equation (7) represents storage cost.

$$C_{3} = \sum_{i=1}^{N_{c}} \sum_{r=1}^{N_{w}} \sum_{s=1}^{N_{q}} (c_{ir}^{w_{2}} + c_{r}^{w_{3}}) X_{irs}^{w}$$
(7)

### Transportation

In the recovery network, product or disassembled unit would be transported between sites. It can be classified into three types of transportation: transportation from collection point i to recovery plant j and

demand site k ( $C_4$ ), transportation between recovery plants ( $C_5$ ), and transportation from recovery plant j to demand site k ( $C_6$ ).

$$C_4 = \sum_{i=1}^{N_c} \sum_{j=1}^{N_f} \sum_{s=1}^{N_g} c_{ijs}^{\alpha} \cdot X_{ijs}^{\alpha} + \sum_{i=1}^{N_c} \sum_{k=1}^{N_d} \sum_{s=1}^{N_g} c_{iks}^{\beta} \cdot X_{iks}^{\beta}$$
(8)

$$C_{5} = \sum_{j_{m}=1}^{N_{f}} \sum_{j_{n}=1}^{N_{f}} \sum_{s=\ell}^{N_{q}} c_{j_{m}j_{n}s}^{\delta} \cdot X_{j_{m}j_{n}s}^{\delta}, j_{m} \neq j_{n}$$
<sup>(9)</sup>

$$C_6 = \sum_{j=1}^{N_f} \sum_{k=1}^{N_d} \sum_{s=1}^{N_d} c_{jks}^{\gamma} \cdot X_{jks}^{\gamma}$$
(10)

#### **Recovery operations**

Each facility performs various recovery operations, such as reuse, repair, recycling, remanufacturing, disassembly, and others. Every operation for an input causes unit operation cost, and this cost has different value depending on the facility's capability. Equation (11) represents operation cost.

$$C_7 = \sum_{j=1}^{N_f} \sum_{p=1}^{N_o} c_{jp}^o \cdot Z_{jp}$$
(11)

#### Penalty for unprocessed /discarded unit at recovery plants

In a recovery plant, some units can be discarded without further processing. Penalty cost for such units is calculated by Equation (12).

$$C_8 = \sum_{j=1}^{N_f} \sum_{s=1}^{N_q} c_{js}^h \cdot X_{js}^h \tag{12}$$

#### Revenue

Besides cost, a recovery network brings about revenue by satisfying customer demand. For example, selling remanufactured products or recovered material returns income for the seller. Equation (13) describes the total revenue of a recovery network.

$$R = \sum_{k=1}^{N_d} \left( \sum_{i=1}^{N_c} X_{iks}^{\beta} + \sum_{j=1}^{N_f} X_{jks}^{\gamma} \right) \cdot r_{ks}^d \tag{13}$$

### 4.4 Constraints

#### Flow balance at collection point

From a collection point, a returned product with state *s* should move to one of the following places: recovery plants, disposal sites, and warehouses. Constraint (14) represents this; here,  $E_{is}$  indicates the total volume of returned product with state *s* at collection point *i*.

$$E_{is} = \sum_{j=1}^{N_f} X_{ijs}^{\alpha} + \sum_{l=1}^{N_g} X_{ils}^{g} + \sum_{r=1}^{N_w} X_{irs}^{w} + \sum_{k=1}^{N_d} X_{iks}^{\beta} \qquad \forall i \in I, \forall s \in S$$
(14)

#### Facility feasibility

A returned product can be distributed only to an available facility. Constraints (15), (16), and (17) constrain this feasibility condition in terms of disposal sites, recovery plants, and warehouses respectively; here,  $\omega$  is an extremely large number.

$$\sum_{i=1}^{N_c} \sum_{s=1}^{N_q} X_{ils}^g \le \omega \cdot Y_l^g \qquad \forall l \in L$$
<sup>(15)</sup>

$$\sum_{i=1}^{N_c} \sum_{s=1}^{N_q} X_{irs}^w \le \omega \cdot Y_r^w \qquad \forall r \in \mathbb{R}$$
<sup>(16)</sup>

$$\sum_{i=1}^{N_c} \sum_{s=1}^{N_q} X_{ijs}^{\alpha} + \sum_{j_m=1}^{N_f} \sum_{s=1}^{N_q} X_{j_mjs}^{\delta} + \sum_{j_n=1}^{N_f} \sum_{s=1}^{N_q} X_{jjns}^{\delta} + \sum_{k=1}^{N_d} \sum_{s=1}^{N_q} X_{jks}^{\gamma} + \sum_{s=1}^{N_q} X_{js}^{h} \le \omega \cdot Y_j^{f}$$

$$j_m \neq j, \forall j \in J$$
(17)

#### Input flow balance at recovery plants

Every input unit of a recovery plant is either from collection points or other recovery plants. Therefore,  $E_{js}$ , the total volume of input unit in state *s* at a facility *j*, is the sum of input flows from collection points and input flow from recovery plants.

$$E_{js} = \sum_{i=1}^{N_c} X_{ijs}^{\alpha} + \sum_{j_m=1}^{N_f} X_{j_m js}^{\delta} \qquad j_m \neq j, \forall j \in J, \forall s \in S$$

$$\tag{18}$$

#### Unit state change feasibility at recovery plants

A recovery operation changes an input's state into another state. This identity change should be feasible. When an operation p uses a unit with state s as its input, the cell (s, p) of the transition matrix has -1 value. The number of operations for the unit in state s cannot exceed the number of inputs with s. Constraint (19) constrains this feasibility condition.

$$E_{js} + \sum_{p=1}^{N_o} Z_{jp} \cdot T_{sp} \ge 0 \qquad \forall j \in J, \forall s \in S$$
<sup>(19)</sup>

#### Capacity of recovery plants

There is a set of operations a facility can do, and the facility can perform only the activities in the set. As for an activity, a facility has the upper bound of input amount, that is, capacity. The facility can deal with only the amount of inputs less than capacity. Capacity for unavailable operation is set as 0.

$$Z_{jp} \le u_{jp}^{f} \qquad \forall j \in J, \forall p \in P$$
(20)

#### Output flow feasibility at recovery plants

The output in state *s* at the recovery plant *j* is equals to the remaining units, changing from the initial input amount due to recovery operation. The output,  $O_{js}$ , increases if the plant *j* performs any recovery operation generating unit with state *s*. In contrast, it decreases if the plant operates recovery operation transforming unit's state into other states.

$$O_{js} = E_{js} + \sum_{p=1}^{N_o} Z_{jp} \cdot T_{sp} \qquad \forall j \in J, \forall s \in S$$

$$\tag{21}$$

#### Output flow balance at recovery plants

An output unit in state *s* should move to either one of other recovery plants or demand sites. Or, a plant could stop to recover the unit even accepting some penalty cost for giving up the recovery. Equation (22) represents this output balance constraints.

$$O_{js} = \sum_{j_n=1}^{N_f} X_{jj_ns}^{\delta} + \sum_{k=1}^{N_d} X_{jks}^{\gamma} + X_{js}^{h} \qquad j_n \neq j, \forall j \in J, \forall s \in S$$

$$(22)$$

#### Demand satisfaction and avoidance of excess fulfillment

Each of demand sites requires an amount of unit in state *s*, and this demand can be satisfied by the input from collection points and recovery plants. This supply of recovered units at the demand site *k* is controlled not to exceed the corresponding demand,  $v_{ks}^{k}$ , by constraint (23).

$$\sum_{i=1}^{N_c} X_{iks}^{\beta} + \sum_{j=1}^{N_f} X_{jks}^{\gamma} \le v_{ks}^d \qquad \forall k \in K, \forall s \in S$$

$$\tag{23}$$

#### Variable condition

Y is a binary variable indicating whether a site opens or not. *X* represents the volume of items moving on the network; thus, every *X* should have nonnegative integer value. Also,  $Z_{jp}$  indicating the number of operation should be nonnegative integer. Constraints (24) and (25) restrain these variable conditions.

$$Y_j^f, Y_l^g, Y_r^w = 0 \text{ or } l \text{ (binary)}$$

$$(24)$$

$$X_{ils}^{g}, X_{irs}^{w}, X_{ijs}^{\alpha}, X_{iks}^{\beta}, X_{jks}^{\gamma}, X_{js}^{\delta}, X_{js}^{h}, Z_{jp} = nonnegative integer$$
(25)

### **5 ILLUSTRATIVE EXAMPLE**

To illustrate the approach, the mathematical model was applied to a simulated cellular phone recovery problem. The parameter values are assigned based on previous literature [23, 24]. Here, we assume that a remanufacturing company collects used cellular phones, recovers them, and resells the remanufactured phones on the market. The company has one main collection point and sells the recovered phones to a recycling center as well as a cell phone market. Also, there is one disposal site and one warehouse available. There are two potential locations for the recovery plants. In sum, this problem has values of i=1; j=2; k=2; l=1; r=1. The site opening cost for plant 1, plant 2, the disposal site, and the warehouse are set as 100,000, 50,000, 10,000, and 50,000, respectively.

As reflected in Table 3, there are 14 possible states that can be taken by a product on the network, s=14. The top seven represent the state of the returned products. Among them, four states are expected to represent the initial state of a returned product: bad function and bad appearance (state 1); good function and bad appearance (state 2); bad function and good appearance (state 3); and good function and appearance (state 4). States 8 through 14 describe the components generated by disassembly. In this case, a cell phone is designed to be dismantled into seven components: housing, antenna, display, microphone, speaker, keyboard, and circuit board. Each of the components can be described by different states; for example, according to its quality. However, in this problem, a component is regarded as a state, and it is assumed that recovery plants do not perform any component maintenance. When a product is disassembled, the resultant components are sent to the market to be sold. The initial inputs, market demand volume, and expected revenues from each market are reflected in Table 3.

State	Descri	ption	$E_{is}$	$v^d_{k_1s}$	$v^d_{k_{2s}}$	$r_{k_1s}^d$	$r_{k_2s}^d$
Product	Function	Function Appearance					
1	Bad	Bad Bad		3000	0	5	10
2	Good	Bad	1100	3000	4750	5	35
3	Bad	Good	1900	3000	0	5	15
4	Good	Good	3300	3000	4250	5	40
5	New	Good	0	3000	3000	5	55
6	Good	New	0	3000	5500	5	45
7	New	New	0	3000	1500	5	60
Component							
8	Hous	sing	0	3000	200	0.25	4
9	Ante	nna	0	3000	200	0.5	4
10	Disp	olay	0	3000	400	1	6
11	Micro	phone	0	3000	400	1	4
12	Spea	lker	0	3000	400	1	4
13	Keyb	oard	0	3000	400	0.25	4
14	Circuit	Board	0	3000	500	1	8

Table 3. States of cellular phone and parameter values

Table 4. Transition matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
							Rep	roces	sing							Ι	Disassembly			
1	-1	-1	-1										-1			-1				
2				-1	-1	-1							1	-1			-1			
3							-1	-1	-1				•		-1			-1		
4			•							-1	-1	-1		1	1				-1	
5	1			1			1			1										
6		1	•		1			1			1									
7			1			1			1			1								
8																1	1	1	1	
9																1	1	1	1	
10																1	1	1	1	
11																1	1	1	1	
12																1	1	1	1	
13													.			1	1	1	1	
14																1	1	1	1	

In this problem, there are 19 recovery operations; i.e., the transition of input units to output units is designed through available operations. As shown in Table 4, operations 1 through 12 represent refurbishing or remanufacturing operations that replace some parts so that an input item is converted to a unit in states 5, 6, or 7. A product can have four possible initial states and regardless of that initial state, a product can be transformed into states 5, 6, or 7. This is represented by first 12 operations. Operations 13, 14, and 15 indicate minor maintenance jobs, such as repair and surface treatment operations, which convert bad states into good states. The transition matrix representing the relationships between states and recovery operations are shown in Table 4. For example, transition 1 describes a recovery operation that refurbishes a cell phone with a bad function and a bad appearance.

After the operation, the cell phone is transformed to one with new functionality and a good aesthetic condition. Operations 16 through 19 describe the disassembly operations. Regardless of the initial states, an item is dismantled into the same set of components represented by states 8 to 14.

Plant 1 is assumed to perform every operation including remanufacturing, but not to deal with minor maintenance in operations 13, 14, and 15. In contrast, Plant 2 conducts minor repair operations. Disassembly operations, operations 16 through 19, are performed at both plants. Plant capacity, the operation cost of each facility, and other unit cost data used here are attached in the appendix.

Using Excel Solver, the optimization problem was resolved and the results are shown in Tables 5 through 8. The objective function value is -183,328. This means that the company can expect maximum revenues of 183,328 when it follows this optimal scenario. First, only recovery plant 1 is open. The collection point sends 3,700 units to the facility in state 1, 1,100 units in state 2, 1,900 units in state 3, and 1,100 units in state 4; meanwhile, the collection point sends 2,200 units in state 4 directly to the cell phone market. After receiving units from the collection point, facility 1 performs several operations, including disassembly for 400 units in state 1. As a result, 3,000 units in state 5, 2,900 in state 6, and 1,500 in state 7 are produced and sent to the cell phone market. Also, all components that resulted from disassembly are transferred to the market: 200 units in states 8 and 9, each, are sent to the recycling center, while the rest are sent to the cell phone market. All  $X_{j_2j_s}^{\delta}$ ,  $X_{j_s}^{\delta}$ , and  $X_{j_{s}}^{h}$ , resulted in 0.

Table 5.	Site	opening	indicator,	Y
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$Y_{j_1}^f$	$Y_{j_2}^f$	$Y_l^g$	$Y_r^w$
1	0	0	0

State	I	Eis	$X_{ils}^g$		$X_{irs}^w$	$X_{ij}^{a}$	r 1s	$X^{\alpha}_{ij2s}$		$X^{\beta}_{ik_{1s}}$	$X_{iki}^{\beta}$	2.5	$X_{j_1k_1}^{\gamma}$	s	$X_{j_1k_2s}^{\gamma}$	X	rγ j2k1s	X	γ j2k2s
1	37	700	0		0	370	)0	0		0	0		0		0		0		0
2	11	00	0		0	110	00	0		0	0		0		0		0	(	
3	19	900	0		0	190	00	0		0	0		0		0		0		0
4	33	300	0		0	110	00	0		0	220	0	0		0		0		0
5		0	0		0	0		0		0	0		0		3000		0		0
6		0	0		0	0		0		0	0		0		2900		0		0
7		0	0		0	0		0		0	0		0		1500		0		0
8		0	0		0	0		0		0	0		200		200		0	0	
9		0	0		0	0		0		0	0		200		200		0		0
10		0	0		0	0		0		0	0		0		400		0		0
11		0	0		0	0		0		0	0		0		400		0		0
12		0	0		0	0		0		0	0		0		400		0		0
13		0	0		0	0		0		0	0		0		400		0		0
14		0	0		0	0		0		0	0		0		400		0		0
						Tab	le 7. N	umb	er o	foperat	tions,	Z <sub>jp</sub>							
Plant	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0	2900	400	0	0	1100	1900	0	0	1100	0	0	0	0	0	400	0	0	0

- · · ·			~	
Table 6.	volume	of product	flow,	, X

Plant	1	2	5	4	5	6	1	8	9	10	Ш	12	13	14	15	16	17	18	19
1	0	2900	400	0	0	1100	1900	0	0	1100	0	0	0	0	0	400	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 8. Cost, revenue, and profit results

$C_I$	<i>C</i> <sub>2</sub>	<i>C</i> <sub>3</sub>	<i>C</i> <sub>4</sub>	<i>C</i> <sub>5</sub>	<i>C</i> <sub>6</sub>	<b>C</b> <sub>7</sub>	<i>C</i> <sub>8</sub>	R	<i>-f</i>
100000	0	0	3000	0	1521.7	197800	0	485650	183328

# 6 CONCLUSION

Product recovery has become a field of rapidly growing interest for product manufacturers as a promising solution for product stewardship as well as economic viability. Product design and recovery logistics design are two major factors of recovery profit. Of critical importance is the fact that those two problems are tightly coupled. Therefore, to maximize the recovery profit, companies should consider product and recovery network design concurrently.

A generic method for optimizing a recovery network design was developed that reflects the impact of product design during network optimization by using a transition matrix. Specifically, the proposed

model regards network design as a set of decision variables that should be optimized simultaneously with the recovery plan. The proposed model derived not only the optimal network design but also the optimal recovery plan for large numbers of products and the expected recovery profit from that plan. This means that the model also performs end-of-life decision making as well as network optimization at the product design stage.

This method contributes to better design solutions to maximize product recoverability. Future work involves applying this method for different product design alternatives, which will help a company to evaluate each alternative in order to select the best design. By simultaneously optimizing product design and network design, a company will be able to identify an optimal product configuration in line with an optimal recovery network.

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### APPENDIX

Table A.1 Cost information																
	$C_{il}^{g_2}$	$c_l^{g_3}$	$C_{ir}^{w_2}$	$C_r^{w_3}$	$c^{\alpha}_{ij_1s}$	$c^{lpha}_{ij_1s} = c^{lpha}_{ij_2s} = c^{eta}_{ik_1s} = c^{eta}_{ik_2s} = c^{\gamma}_{j_1k_1s} = c^{\gamma}_{j_1k_2s} = c^{\gamma}_{j_2k_1s} = c^{\gamma}_{j_2k_2s} = c^{\gamma}_{j_mj_ns}$									$c_{j_2s}^h$	
State	e Disposal Stora		rage	Transportation										Penalty		
1-7	0.1	0.5	0.1	0.5	0.3	0.1	0.1	0.3	0.2	0.2	0.1	0.2	0.3	0.5	0.5	
8			-						0.04	0.04	0.02	0.04	0.04	0.1	0.1	
9									0.004	0.004	0.002	0.004	0.004	0.1	0.1	
10									0.004	0.004	0.002	0.004	0.004	0.1	0.1	
11			-						2E-04	2E-04	1E-04	2E-04	2E-04	0.1	0.1	
12			-						0.002	0.002	0.001	0.002	0.002	0.1	0.1	
13			-						0.02	0.02	0.01	0.02	0.02	0.1	0.1	
14									0.034	0.034	0.017	0.017	0.034	0.1	0.1	

### Table A.2 Plant information

								-				-							
Plant	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	Capacity																		
1	3k	3k	3k	3k	3k	3k	3k	3k	3k	3k	3k	3k	0	0	0	3k	3k	3k	3k
2	0	0	0	0	0	0	0	0	0	0	0	0	3k	3k	3k	3k	3k	3k	3k
Operation cost																			
1	37	30	40	12	5	15	35	30	40	10	5	15	0	0	0	2	1.5	2	1.5
2	0	0	0	0	0	0	0	0	0	0	0	0	25	2	25	1.5	1	1.5	1

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