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FEM MODEL FOR THE ANALYSIS OF ROTATIONAL SPEED INFLUENCE ON TOOTH CONTACT PRESSURE DISTRIBUTION OF THIN-RIMMED GEARS WITH ASYMMETRIC WEB ARRANGEMENT

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1. Introduction

The designer is very often restricted by increased demands requiring that mechanical elements should carry high loads at high speeds with both size and weight kept to minimum. A common design goal for gears used in power transmission is to reduce weight and size. To achieve this goal, some gear designs incorporate thin rims. The application of thin-rimmed gears with asymmetric web arrangement is required primarily in compact gear boxes in relation to which with light weight and small space are necessary, e.g. helicopter and turboprop high-power transmissions. This makes it necessary for gear designers to know the real local stress state in tooth root and contact pressure at teeth flanks. This data is also used to investigate gear failure mechanisms such as surface pitting of gear teeth flanks, and tooth or rim breakage in tooth root. The most common tooth root stress and tooth contact pressure calculation methods are based on conventional standard procedures, published by German Institute for Standardization (DIN), American Gear Manufacturers Association (AGMA), International Organization for Standardization (ISO) and Japanese Gear Manufacturers Association (JGMA). However, they are limited to stress calculation in the tooth root and to tooth contact pressure calculation of a solid gear and are not fully applicable for analyses in concern to thin-rimmed gears because they are based on two-dimensional plane stress or plane strain model and do not consider thin-rimmed gear design factors such as: web thickness, web arrangement, gear radius and gear rotational speed. An exception can be found in AGMA and ISO, where, based on the conventional approach, a stress modifying factor for thin-rimmed gears is proposed, however, it considers only rim thickness and nothing else. A number of researchers have investigated the effects of various design parameters on gear stresses. These studies included analysis using the finite element method (FEM), complex potential method, strain gauge and photoelastic experiments. In the beginning, FEM stress analysis of thin-rimmed gears was limited to two-dimensional analysis [Oda 1981], [Bibel 1991]. After that, three-dimensional partial deformation models that were used for a solid gear stress analysis were also used for a thin-rimmed gear [Abiy 1995]. Three-dimensional analysis is required to perform simulation of influence of gear web arrangements on stresses [Opalić 1999]. Loaded tooth contact and stress analysis on the basis of the whole gear model showed that partial deformation model is not suitable for thin-rimmed gears if the rim is fixed on the boundary [Li 2002], [Conrado 2007]. The effect of rotational speed on crack propagation in a thin-rimmed gear with symmetric rim support arrangement was explored [Lewicki 2001]. In short, it is concluded that tooth root stress and tooth contact pressure calculation problems of a thin-rimmed gear that have special design have not been solved. A numeric determination of design parameters of a thin-rimmed gear requires a demanding approach relating to stress analysis and conditions of tooth engagement during operation. Current numerical methods are available for the solution of the elasticity problem for complex domain so it is possible to calculate accurately the local tooth root stress and contact pressure at teeth flanks. The objective of the article is to use advanced engineering tools for a numeric calculation of thin-rimmed gear that have a special design and operating conditions. Presented parametric FEM model uses a true geometry three-dimensional gear model. Once created, designers can evaluate effects of applied torque, web arrangement, web-thickness, rim-thickness and rotational speed on tooth contact pressure distribution by simply changing parameter values inside the FEM software. A practical application is shown on a thin-rimed gear with asymmetric web arrangement engaged with a solid spur gear and the influence of rotational speed on tooth contact pressure distribution is evaluated. Tooth root stress was evaluated as well, however, due to limited space, detailed results are not published.

2. Gear tooth profile

The gear tooth profile generated by a rack-form type of cutting tool, such as hob, can be divided into four sections: top land, involute section, trochoid section and bottom land. In order to obtain very accurate gear tooth profile geometry with the resolution of 1 μm and to make its generation automatic, a parametric computer program has been developed. On the basis of provided gear geometry parameters, it calculates coordinates of each of four stated tooth sections using state-of-the-art mathematical formulation [ISO 2006]. The output data, containing two-dimensional coordinates of single tooth sector is stored in a text file. This data can be read by any available FEM software and required geometry can be created within it instead of being imported from other Computer Aided Design (CAD) packages as geometry importation tools may cause a lot of problems, primarily due to disparity between standard interpretation and level implementation [Mao 2007].

3. FEM model of gears in contact

The commercially available FEM software Abaqus version 6.7.1 has been used for analysis. Pre- and postprocessing are done in Abaqus/CAE and numerical analysis in Abaqus/Standard. The calculated coordinates of tooth profile are imported into the FEM software by using the Python script. A spline curve is used to connect a series of points, and complete two-dimensional gear geometry is generated using a radial pattern of a single-tooth sector. The three-dimensional gear model is obtained through extruding the two-dimensional geometry. Each tooth sector is partitioned into eight six sided solid partitions so that a structured meshing technique could be applied and elements of good quality generated. The mesh is refined around the mating teeth in order to accurately calculate the local tooth root stress state. Two additional solid partitions around the geometrical contact line were created on each mating tooth, so mesh could be additionally refined and true tooth contact pressure distribution calculated. Partitioning process and mesh refinement are parametrically controlled. Incompatible modes hexahedral linear elements are used for the mesh. These types of elements are suggested for the convergence of contact algorithm and the elements are enhanced by incompatible modes to improve their bending behaviour. The incompatible mode elements perform almost as well as second-order elements in many situations if the elements have an approximately rectangular shape. Because of the 13 added internal degrees of freedom due to the incompatible modes, these elements are somewhat more expensive than regular first-order displacement elements; however, they are significantly more economical than second-order elements [Abaqus 2007]. Element tests were conducted on partial deformation solid gear model, with uniform load applied at the highest point of single tooth contact (HPSTC). The incompatible modes hexahedral linear elements gave results that match the results of second-order elements. The material is considered homogenous and isotropic with a linear elastic behaviour. Small displacement hypothesis is assumed for the purpose of the analysis. The gear pair contact position is chosen to load the thin-rimmed gear (gear wheel) at the HPSTC as the largest load and maximum contact pressure point. The pinion position, with respect to the gear wheel, has been calculated with the resolution of 1 μm. The contact between mating teeth is considered a dry frictionless contact, assuming small sliding between them. The contact constraint is enforced with a

Lagrange multiplier representing the contact pressure in a mixed formulation. A thin-rimmed gear has to be modelled as the whole gear deformation model while the mating solid gear, in order to reduce computational costs, can be modelled as a partial deformation model having at least five teeth. When two thin-rimmed gears are engaged, both of them have to be modelled as the whole gear deformation model. Kinematic coupling constrain is used in order to prescribe a torque to a wheel gear web inner cylindrical surface without constraining the cylindrical displacement, but constraining the radial and the axial ones on the same surface. Rotational body force is applied only to the thin-rimmed gears due to small influence on solid gears. To impose kinematic and static boundary condition, the mating gear (pinion) inner cylindrical surface is fixed. If a solid gear is modelled as a partial deformation model, two radial planar surfaces are also fixed. The validation of the proposed FEM model was done by calculating the maximum tooth contact pressure for two solid spur gears in contact and by comparing it with the ones obtained by Hertzian's formula. Both gears had the same parameters as indicated in Table 1.

Table 1. Parameters of gears

Number of teeth	$z_1 = z_2$	50
Module	m	4 mm
Pressure angle	α	20°
Shifting coefficients	$x_1 = x_2$	0 mm
Edge radius of cutter	ρ_{fP}	$0.215m$ mm
Young's modulus	$E_1 = E_2$	210 GPa
Poisson's ratio	$v_1 = v_2$	0.3
Mass density	$\rho_1 = \rho_2$	7800 kg/m^3
Loading torque	T	294 Nm (30 kgm)
Rotational speed	n	2000, 4000, 6000, 8000, 10000 rpm

Uniform tooth contact pressure distribution, as is shown in Figure 1, was obtained on the basis of the FEM simulation and the maximum tooth contact pressure was 398.5 MPa. This correlates well to a value of 408 MPa for maximum contact pressure obtained from Hertzian's formula for the plane strain. Also maximum principal stress at tooth root was calculated and magnitude of 53.8 MPa matched the one of 53.65 MPa based on conventional standard procedures described in ISO. Calculations were performed without considering the influence of the rotational body force.

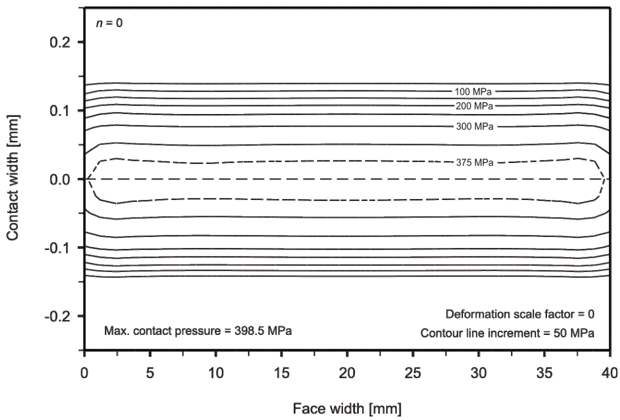

Figure 1. Contact pressure contour lines for solid spur gears engaged

In order to validate the solution using the rotational body force option of FEM software, 3D disk model [Lewicki 2001] was analyzed. Results showed excellent agreement with the theoretical ones.

4. Practical application

In order to obtain design modification factors for thin-rimmed gear designs studies should be conducted on a model that properly provides the minimum practical elastic support for thin-rimmed gears (Bibel 1991). Following that, the gear pair (Li 2002) shown in Figure 2 is used as an example for the calculation and FEM model evaluation. The suggested thin-rimmed gear has asymmetrically arranged thin web as a support and is, therefore, more easily deformed by rotational body force than a gear with a symmetrical one. The mating gear of the thin-rimmed gear is a solid gear with the same parameters as indicated in Table 1. Gears are error free.

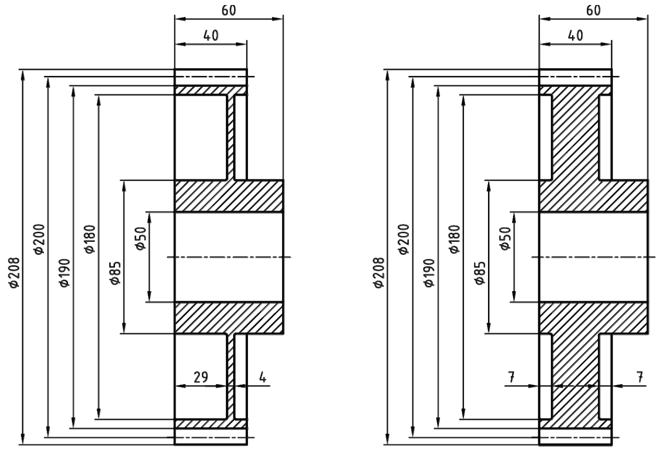

Figure 2. A pair of spur gears

The thin-rimmed gear FEM model is partitioned into 404 regions so that a structured meshing technique could be applied, for the same reason the solid gear partial deformation model is also partitioned into 34 regions.

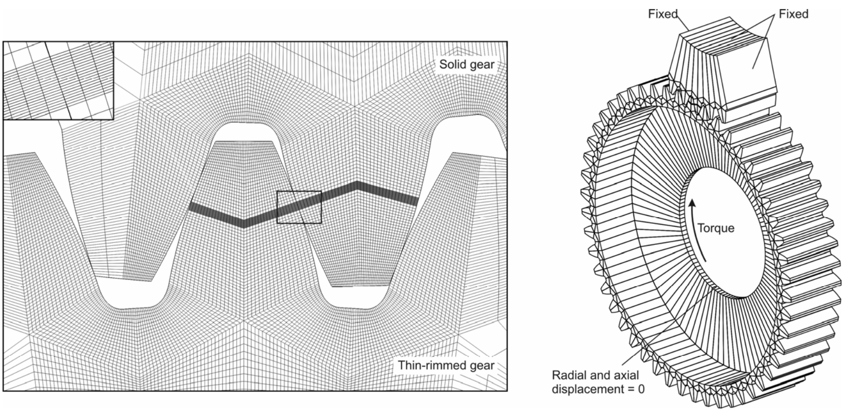

Figure 3. Model used in FEM analysis along with refined mesh detail

Both parts are meshed with the same global mesh size factor of 1 mm. The mesh is refined around the mating teeth with the local mesh size factor of 0.15 mm, and especially near the geometrical contact line at HPSTC with the factor of 12.5 μm, as is shown in Figure 3. The total number of elements is

806818 and the number of nodes is 893986. The three-dimensional model of engaged gears used in the analysis along with applied torque and boundary conditions is shown in Figure 3. Calculations were conducted for six cases. The first case included applied torque of 294 Nm (30 kgm) without body force load [Li 2002]. Other cases included applied torque of 294 Nm and rotational body force load due to gear rotational speeds at 2000, 4000, 6000, 8000 and 10000 rpm. The same calculation is repeated for a thin-rimmed gear with increased rim thickness.

5. Results and discussion

It has been found that the largest magnitude of contact pressure is at the beginning when there is no gear rotation, only torque is applied. As the rotational speed is increased, the magnitude of max. contact pressure is decreased, and is the lowest at 10000 rpm. This happens because rotational body force opens the thin-rimmed gear, as is shown in Figure 4, by pushing the free end of the rim toward the mating gear and thus providing the additional support for the mating gear tooth. Now the load acting on tooth is distributed on a larger surface and the magnitude of max. tooth contact pressure decreases.

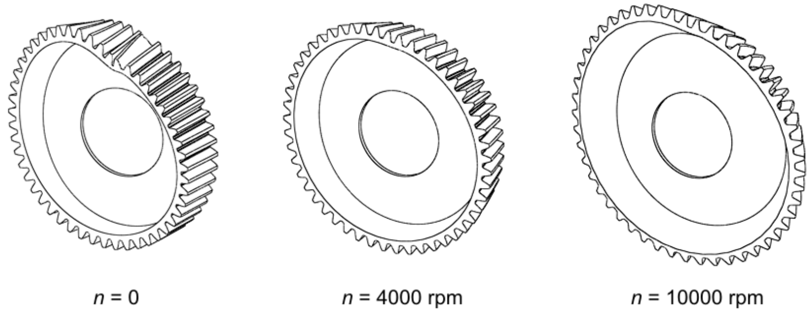

Figure 4. Thin-rimmed gear displacement (scaled) at 0, 4000 and 10000 rpm

It has also been established that the tooth contact pressure distribution is changed from a side heavy contact into an almost uniform contact with inclined maximum pressure distribution curve, for the same reason. Figure 5 contains plots of tooth contact pressure contour lines for rotational speeds of 0, 4000 and 10000 rpm combined with the torque of 294 Nm applied at a thin-rimmed gear web. Special attention must be paid to tooth root max. principal stress, because its largest magnitude is at 10000 rpm. Close look at Table 2 with summarized data reveals that magnitude of tooth root max. principal stress is decreased while rotational speed is increased from 0 to 2000 rpm, however, at a certain speed between 2000 and 4000 rpm this trend changes and the magnitude of toot root max. principal stress is increased along with the rotational speed.

Table 2. Summary of calculated results for gear with rim thickness of 5 mm

Solid gear (pinion) engaged with the: →	Thin-rimmed gear with rim thickness of 5 mm						Solid gear
Gear rotational speed [rpm]	0	2000	4000	6000	8000	10000	0
Contact pressure [MPa]	565.7	533.2	499.1	464.4	429	408	398.5
Contact pressure modification factor	1.42	1.39	1.25	1.17	1.08	1.02	1
Max. principal stress at tooth root [MPa]	105.6	104.9	181.8	267.8	353.8	439.9	53.8
Tooth root stress modification factor	1.96	1.94	3.38	4.98	6.58	8.18	1

This is also an after-effect of rim deformation because the gear rim is compressed toward the gear centre due to the torque applied at the beginning. However, as the gear rotational speed is increased, the rim is deformed in the opposite direction due to the rotational body force and thus, the stress state at the tooth root is changed. Modification factors are obtained by dividing magnitude of max. contact pressure and max. principal tooth root stress for thin-rimmed and solid gear engagement with the appropriate one for two solid spur gears engagement.

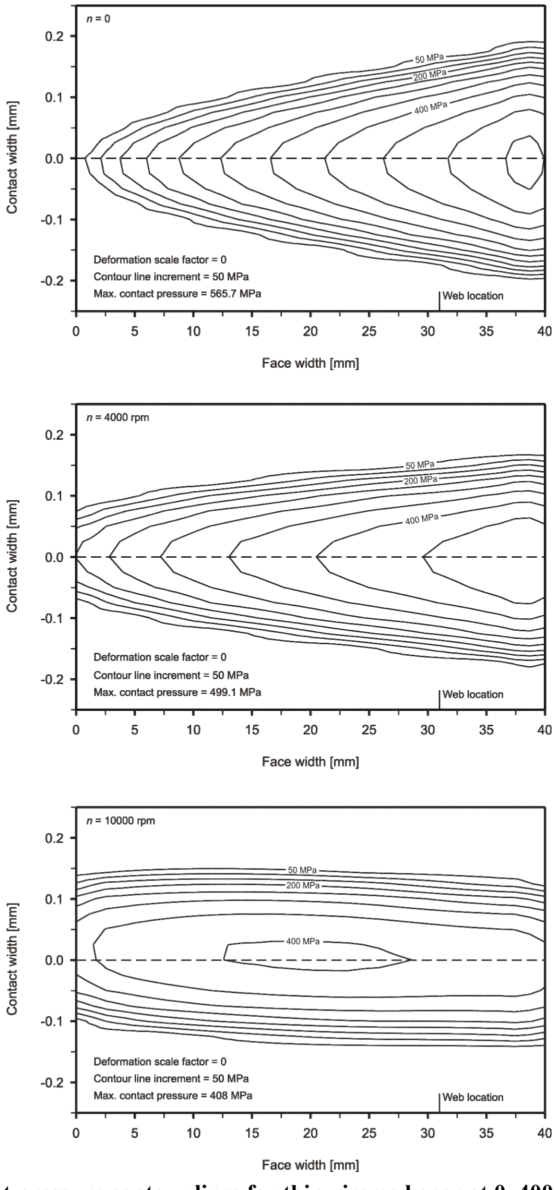

Figure 5. Contact pressure contour lines for thin-rimmed gear at 0, 4000 and 10000 rpm

If calculation methods for solid spur gears are used, calculated values must be modified with the appropriate modification factor. Stress modification factor value of 1.96 at n = 0 is in good agreement with the reference value 1.92 [Li 2002]. One of the ways to lower the max. tooth contact pressure and max. tooth root stress as well is to increase rim stiffness by increasing rim thickness. By simple changing the gear rim thickness design parameter in FEM model from 5 to

9 mm, influence of gear rotational speed is evaluated for a new, redesigned gear. Automatic remeshing is done with the same parameters as in the previous example and summary of calculated results is shown in Table 3.

Table 3. Summary of calculated results for gear with rim thickness of 9 mm

Solid gear (pinion) engaged with the: →	Thin-rimmed gear with rim thickness of 9 mm						Solid gear
Gear rotational speed [rpm]	0	2000	4000	6000	8000	10000	0
Contact pressure [MPa]	548.5	516.1	480.8	444.1	410.1	408.2	398.5
Contact pressure modification factor	1.38	1.30	1.21	1.11	1.03	1.02	1
Max. principal stress at tooth root [MPa]	85.0	84.4	139.8	203.8	268.7	333.9	53.8
Tooth root stress modification factor	1.58	1.57	2.60	4.29	4.99	6.21	1

Due to the thicker rim, the gear weight is increased by 25% and tooth root stress is decreased by between 19 and 24%, however the max. tooth contact pressure is decreased by no more than 5%. The rim stiffness can also be increased if stiffer support geometry is used [Bibel 1991], [Opalić 1999] e.g. thicker web end/or smaller web offset from centre, however, this will also result in increased gear weight and/or changes in gear geometry. The calculated side heavy tooth contact pressure distribution, for both rim thickness examples, at gear rotational speeds fewer than 8000 rpm can cause surface pitting of gear teeth flanks and gear may fail. If asymmetric web arrangement is requested, than micro-geometry modifications, such as gear tooth flank lead correction and/or crowning, are suggested in order to lower the max. tooth contact pressure providing a longer gear life. This way the gear weight does not have to be increased and gear macro-geometry remains unchanged. Recommendations for the gear micro-geometry design can be made on the basis of calculated tooth contact pressure distribution and its magnitude for appropriate working gear rotational speed.

6. Conclusion

In recent years, compact size, low weight and high speed have become important considerations in gear design. This makes it necessary for gear designers to know the real tooth root stress and tooth contact pressure. Conventional standard procedures are limited to stress calculation in the tooth root and to tooth contact pressure calculation of a solid gear and are not fully applicable for analyses in concern to thin-rimmed gears. A three-dimensional parametric finite element solid model capable to calculate real tooth root stress and tooth contact pressure of complex gear designs is presented in this paper. The 3D model is generated within FEM software; it is not imported from other CAD. Good mesh element quality is achieved through parametric partitioning which allows structured meshing. In this way, the use of incompatible modes hexahedral linear elements for the mesh could be presented. These elements have good contact and bending behaviour, so tooth root stress and tooth contact pressure could be calculated at the same time. The calculation time and the hardware needed are significantly lower than for the second-order ones, but yield similar results. The parametric model along with the automated meshing process allows simple changes in thin-rimmed parameters. In this way, a new FEM model can be prepared within few minutes. Detailed analysis of operating conditions represents a more precise approach to numerical determination of stress conditions, and consequently it makes more appropriate and better design solutions possible. The influence of gear rotational speed on tooth contact pressure distribution has been evaluated for a thin-rimmed gear with asymmetric web arrangement engaged with a solid spur gear. It has been found that the gear rotational speed has great influence on maximum principal tooth root stress at speeds exceeding 2000 rpm. At 10000 rpm it is over 4 times larger than in relation to a gear that is not rotating and over 8 times larger than in relation to two solid gears being engaged. It has also been established that the increase in the gear rotational speed has a beneficiary effect on tooth contact pressure. Its magnitude is largest when a gear is not rotating and lowest at 10000 rpm. Tooth contact pressure distribution on a thin-rimmed gear is changing significantly from heavy side contact when the gear is not rotating to the almost uniform contact at 10000 rpm. From the stated, it can be concluded that rotational speed influence must be included in strength calculations for thin-rimmed gears with asymmetric web arrangement. Increase in rim and/or support geometry stiffness will lower the max. tooth root stress and max. tooth contact pressure, however, this will result in increased gear weight and/or change in gear web arrangement. When a lightweight gear design with asymmetric web arrangement is required, presented FEM model is an effective tool that can evaluate whether gear design is satisfactory or not, due to different tooth contact pressure distribution at different gear rotational speeds. Also, recommendations for the gear micro-geometry design can be made on the basis of calculated tooth contact pressure distribution at working gear rotational speed. The recommendations can be used for thin-rimmed gear redesign and manufacturing. In this way surface pitting can be reduced and final designed life extended. At present state, the proposed FEM model is not capable to evaluate the influence of gear micro-geometry modifications, however, this model improvement is planned for the future. The described approach of numeric determination of tooth root stress and tooth contact pressure conditions represents thorough and detailed analysis of actual operating conditions used to describe load cycles associated with tooth engagement. This forms the basis for defining actual parameters of fatigue, crack initiation and, consequently, life-cycle of gear.

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