Abstract

In the fierce market competition, early and reliable cost estimation gains a central position for the survival and expansion of companies involved in manufacturing power transmission aggregates. In this context, a method for the estimation of the manufacturing costs for newly designed or prototype shafts will be presented. Thus, design principles in combination with endurance, deformation and vibration constraints will be used in order to conduct the cost forecast in the early development phases. Besides, through the application of Gauss’s systematic error propagation formula, an error-tolerance value will be provided for the cost estimation. The method is implemented in a software module so that prompt estimations can be performed, while the definition of cost optimization objectives will be facilitated through a sensitivity analysis.

Keywords: Design Methods, Techniques, and Tools

1. Introduction

The difficult conditions of international competition impose on companies the minimization of product disposition price, the maximization of the quality provided and simultaneously the minimization of development and production time [1, 2, 3, 4]. The acceleration of the development procedures leaves product developers less time to think and urges them sometimes to take hasty decisions [5]. Moreover, during the product development phase the major quality features and the allocation of almost 70-80% of the product manufacturing costs are determined [2, 3] while the development time often account for the lion’s share of the total product realization time [1, 2]. Thus, it is of crucial importance to provide engineers with methods and tools based on model-patterns of product and process [6], which will help them to calculate the product costs already in the early development phases and take the right decisions.

Shafts belong to the central elements for power transmission. Besides, regarding their manufacturing costs, shafts usually belong to the group of the most expensive machine components (for characteristic examples see [2, 3]).

A method, which allows the estimation of manufacturing costs for new designed or prototype shafts in the early concept phase while indicating the systematic error of the calculated value, is presented. Besides, the method allows the performance of a cost sensitivity analysis concerning design factors. The estimation is using functional and economical features based on the requirements list, considering shaft endurance, deformation and vibration conditions. Hence, an early cost estimation and optimization of a shaft is possible while the development time can be reduced.
2. Shaft cost estimation fundamentals

Naturally, it would be desirable that the costs of a product like shafts could be precisely calculated directly after the clarification of the Product Design Specifications (PDS). Unfortunately, the fact that imprecision is an integral part of engineering design [7] allows only a gradual cost estimation with increasing accuracy while proceeding from the conceptual to the embodiment and detail design phases, since the design variables obtain concrete ranges and values [7, 8, 9] within this process. Existent estimating methods (see [2, 3, 10, 11, 12, 13]) can provide reliable estimations for products and systems in the early development phases, but only when critical preconditions are fulfilled (which prohibit a broad implementation of those methods). Especially for individual machine components, it seems that parametric equations can lead to reliable estimations in the concept phase [3, 11, 12, 14, 15], since they correlate functional and behavioural variables (properties and relationships) of the component and the corresponding manufacturing processes [16]. For individual components with particular characteristics, such variables can be “extracted” through abstraction of main relevant geometrical, structural and material attributes. Then, through a single or multi-parameter regression analysis, component cost formulas can be developed, which sometimes enable the direct use of PDS data for cost estimation [14, 15]. The weak point of such regression formulas is that they yield reliable cost values only for components with similar characteristics (regarding load conditions, form kinds and application cases). Furthermore, they need a nontrivial amount of data referring to past products [2, 14, 15]. This means that with increasing uniqueness-differentiation grade of a new design or prototype in relation to past products, the estimations’ reliability and precision increasingly decreases. Besides, such functions have a limited validity, since they are company-dependent, and hence are not transferable.

On the other hand, although general applicable equations based on the Dimensioning Theory (Bemessungslehre, see [17]) utilize physical functional relationships for forming cost parametric equations, they usually omit vital design parameters and they depend strongly on the users' estimation potential, which has a direct influence on the result reliability. Regarding shafts, the VDI-2225 Guideline [17] introduces two equations for the calculation of the manufacturing costs of prototype shafts considering separately its load-carrying capacity and its deformation concerning deflection and rotating angle. Deflection angle and vibration behaviour of the shaft are not taken into consideration. This calculation is solely presented for shafts with a central bending force or a torsional moment. A model for the estimation combining both loads is not included. Thereby, a maximal shaft diameter can be determined. This diameter is characteristic for a rod-shaft, because material costs are calculated from the rod volume multiplied by a material cost rate. Manufacturing costs (usually regarding turning) are estimated through the multiplication of an unique cost rate and the material volume to be removed by the process. Nevertheless, this cutting volume should be estimated by the user and thus, it is a strongly subjective factor.

Although cost parametric equations generated from both regression techniques and the dimensioning theory allow principally the performance of cost sensitivity analysis regarding design and process parameters, this option is usually neglected. Nevertheless, this is a significant feature for the design of cost optimized components [18].

In order to resolve these problems and with the consideration of the general prerequisites for a forward cost estimating process [11], the method for the implementation of parametric cost estimation for prototype shafts will be based on their functional and behavioural properties interrelated with manufacturing factors. Therefore, it is necessary to localize appropriate parameters and relationships of the embodiment and detail design, which will be logically abstracted and adapted in a framework for an estimation equation in the early phases. For the
determination of such variables and relationships, procedures of the phase model for product development of Pahl and Beitz as well as the proposed design principles should be used [8].

For the abstractions necessary (mainly concerning shaft topologies), principles of the geometrical pattern matching will be implemented [19]. Besides, for the cost equation structuring on a mathematical basis, the mindsets of the Axiomatic Design Theory [20] can be used (concerning the mapping between functional requirements of the functional domain, design parameters of the physical domain and process variables of the process domain). At this stage, the lack of establishing the functional parameters from customer needs [21], in other words the connection with the PDS, can be compensated through the utilization of the elements of the transformation method of Kusiak for mapping and coupling design requirements with functions in the concept phase [22].

Thus, based on the requirements list data for the prototype shaft, design specifications, geometry parameters and shaft topologies will be identified. Concurrently, the possible manufacturing processes will be considered. Then, while regarding design principles, endurance, deformation and vibration constraints will be involved in the procedure. Thereby, through an appropriate mathematical processing, the shaft shape before and after the manufacturing processes will be approximated. This is significant for the cost estimation. Thus, the application of partial material and manufacturing cost rates as well as the utilization solely of design specification data of the requirements list will enable the shaft cost estimation in the concept phase.

Besides, the parameter values of the requirements list can be accompanied by an error indication so that the tolerance for the cost estimation value as well as for critical intermediate parameters on each stage throughout the method evolution will be specified. Gauss’s systematic error propagation formula will be implemented for the error estimation.

3. Method

3.1 Method limitations

Initially, some basic assumptions must be clarified in order to define the limitations of the method. Hence, the method will apply for ductile materials, which are considered homogeneous, isotropic, with linear-elastic properties without internal damping. For the methods’ presentation the most common “rectilinear” shaft topologies will be regarded. A shaft topology here describes the type and the arrangement of the factors resulting to a shaft geometry. These can be gained from the abstraction of shaft characteristics on the embodiment or detail design phase and projected as working structures (Wirkstrukturen, see [8]) on the early concept phase. Initially, such factors are regarded to be the main external load (Forces or Moments) and the bearing forces as well as their arrangement on the shaft (Figure 1). Concerning the dimensioning methods, endurance (bending and torsional alternating load), deformation (deflection, deflection and rotating angle) and vibration (bending and torsion vibration) will be regarded. However, several – usually secondary – parameters will be ignored in a first instance: the shaft dead weight, axial load, possible eccentricities and unbalances, power losses (i.e. of tribological nature), the influence of special temperature and corrosive environments, the influence of the manufacturing processes on the load carrying capacity of the shaft, the effect of the bearing resilience as well as shear deformation and axial vibration. In special cases they must be taken into account either by using a proper variable or by regarding additional equation(s) in the method’s structure. Concerning the costing method that of the short differentiating job order costing [2] will be used. For the calculation of the primary machining times, only the typical formulas for the respective processes will be utilized (see [2, 3, 8, 12]).
Generally, it is aimed at using a limited but indispensable set of model parameters for the partial methods based on the requirements list. These parameters consist of data including provisional geometry, provisional load, material, design threshold values, manufacturing data and cost rates. The parameters will be indicated in each method presentation stage.

### 3.2 Shaft form approximation

In a first stage, a shaft topology must be chosen. For the presentation of the method the calculations concerning the topology “power input-output central” (see Figure 1) will be used. Then, the load distribution on the shaft concerning alternating bending (caused by the bending forces $F_L$ and $F_R$ in N) and torque (caused by the power $P$ in W to be transmitted by $n$ rotations in 1/s) must be calculated (see Figure 2, $z$ is the clumping factor).

\[
M_{t_{\text{max}}} = I_L F_{BL} \quad \text{or} \quad I_R F_{BR}
\]

\[
M_{t_{\text{max}}} = 30^2 P / (\pi \cdot n)
\]

\[
F_{BL} = F_L / (L - L') + F_R / (L - L') / (z \cdot L)
\]

\[
F_{BR} = F_R / (L - L') + F_L / (L - L') / (z \cdot L)
\]

For the parameters an error or range value may be given. This value will be used to calculate a tolerance for each parameter, whose calculation formula includes the error/range indicated variable (i.e. see Figure 3). This feature makes the determination of the range for the shaft cost value possible. Besides, this allows the consideration of the vagueness of some parameters in the concept phase as well as their influence on the overall cost value.

\[
\sum \left( \frac{\partial f_i}{\partial x_j} \right)^2 \cdot f_i^2 = \sum \left[ \frac{\partial G}{\partial x_j} \right]^2 \cdot f_i^2
\]

\[
M_{t_{\text{max}}} = 30 \cdot P / (\pi \cdot n)
\]

\[
f_{M_{t_{\text{max}}}} = \sqrt{\left( \frac{30}{\pi \cdot n} \right)^2 \cdot f_P^2 + \left( \frac{30 \cdot P}{\pi \cdot n^2} \right)^2 \cdot f_n^2}
\]

In a similar way, a sensitivity analysis regarding each construction parameter can be performed (meant as the measure of the influence that the variation of a factor has on the manu-
facturing costs) through the consideration of the differentials of chosen factors in the corresponding load, geometry and cost functions.

The maximal possible shaft diameter ($d_{\text{max}}$ in mm, see Equation 1) concerning the shaft endurance can be calculated from the shaft rough calculation formula of Niemann [23] while regarding simultaneously maximal bending and torsional stresses according to the Distortion Energy Theory. In Equation (1), $\sigma_{bw}$ stands for the flexure fatigue strength of the material in N/mm², $S_D$ is a failure safety factor, $\beta_{kb}$ stands for the notch factor for an estimated notch at the region of the maximal load, $b_0$ and $b_1$ are the bulk and the surface factor concerning the shaft size and the influence of the surface quality on the endurance respectively (see [23]).

$$d_{\text{max}} = \frac{3}{32} \cdot \sqrt[3]{\frac{M_{b_{\text{max}}}^2}{\pi \cdot \sigma_{bw}}} \cdot \left( S_D \cdot \frac{\beta_{kb}}{b_0 \cdot b_1} \right)$$  \hspace{1cm} (1)

In order to approximate the shaft shape, the design principle of uniform strength (see [8]) will be used. This principle states that for an optimal design the allowable bending stresses along a shaft (quotient of the bending moment and the shaft section modulus along the shaft) should be constant. This means that, for instance, the bending stress at the point of maximum bending moment $M_{b_{\text{max}}}$ has to be equal to each bending stress $M_b(x)$ along the shaft ($x$-coordinate). Through this equation we can take the theoretical optimal shaft diameter as function of the shaft length $d(x)$ (see Equation 2) according to the principle of uniform strength. In this case the “shaft core” should look like that in Figure 4. In order to consider the influence of the torque, the $d_{\text{max}}$ in the Equation (2) corresponds that of the Equation 1. It must be emphasized that for the shaft core only the absolute values of the bending moments are significant.

$$d(x) = d_{\text{max}} \cdot \sqrt[3]{\frac{M_{b_{x}}(x)}{M_{b_{\text{max}}}}}$$  \hspace{1cm} (2)

This shaft core must be enveloped from the “real” shaft shape, which consists of several stepped sections. In order to approximate the end-volume of a shaft based on this core, the tangents $D_L$ and $D_R$ (Figure 4) of the half shaft diameter function $d(x)$ on the load locations must be set up (Figure 4, note that the tangent equations give the half diameter along the $x$-coordinate and therefore they must by doubled in order to give the “shaft diameter”. See the shaft fixed coordinate system!). The function of the bending moment distribution is not continuous having as result that the $d(x)$ function is continuous only in sections. Therefore, the tangents should be considered for appropriate $d(x)$ segments at locations short before or after the discontinuities at the load locations. For example analysed, the $d(x)$ segments on the left and on the right of the load locations are chosen. The intersection point of the tangents themselves ($x_{D_{\text{max}}}, D_{\text{max}}$) as well as their expansion limit defined from the nominal shaft length at the points $((L-L_0)/2, D_{\text{minL}})$ and $((2L+L_0)/2, D_{\text{minR}})$ determine segments, whose rotation around the shaft axis generates conical solid bodies. It is assumed that the thus defined shaft volume approximates that of the “real” end-shaft with stepped sections (see Figure 4).

Basically, the number of stepped sections of the “end-shaft” influences the secondary and set-up time regarding the cost calculation. Their number can be estimated considering the application (i.e. combustion engine, small pumps, transmissions, etc.) as it is a function of characteristics depending on the application system which a shaft belongs to. In the example regarded and under the assumption of a gear shaft, the number of the stepped sections can be
“roughly” estimated to 9-11: 2 for the bearings, 2 for the seals/strip-rings, 2 for the load sections, 1-2 for the bearing fixture, 1-2 for the fixture of the load transmission components and 1 for the section between the load sections. The length of the stepped sections depends on the dimensions (width) of the components to be coupled with the shaft and from the form of the “shaft-core”.

The maximum shaft diameter of the “cone-shaft” determines the diameter of the rod, corresponding the shaft row material shape and is significant for the material costs. The volume difference between the rod and the cone-shaft it is the material volume to be removed by the cutting process and it is significant for the manufacturing costs.

However, beside endurance, deformation and vibration constraints must be considered. To do so, based on the cone-shaft, a simple rod-shaft with equivalent moments of inertia is formed (diameter $D_{eqB/P}$, B stands for area and P for polar moment of inertia). For example, a rod with the diameter $D_{eqP}$ of the Equation (3) has an equivalent polar moment of inertia to that of the cone shaft. “k” is a factor, which considers the influence of the load arrangement on the deflection of the (double-cone) shaft.

$$D_{eqP} = 2 \cdot k \left[ \int_{l_a}^{x_{max}} D_L^4(x) \frac{dx}{L} + \int_{x_{max}}^{(L-l_a)} D_R^4(x) \frac{dx}{L} \right]$$

Then, based on the failure criteria concerning allowable deformations (deflection, deflection and rotating angle, see [23]) as well as vibration (critical ranges for bending and torsion, see [23]), allowable diameters of simple rod-shafts are determined ($D_{fX}$, X stands for a failure criterion). For example, Equation 4 gives the minimum rod diameter $D_{\theta_a}$, which complies with the criterion of the allowable rotating angle $\theta_a$ ($\theta_a$ in Rad, $G$ is the shear modulus in N/mm²).

Then, these diameters must be compared with the equivalent cone-shaft diameters. If the value any of the allowable diameters exceed those of the equivalent, then the values of $D_{max}$, $D_{minL}$ and $D_{minR}$ (see Figure 4) must be amplified by their difference (i.e. if $D_{\theta_a} > D_{eqP}$, then $D_{maxNEW} = D_{max} + D_{\theta_a} - D_{eqP}$). Practically, that means that the tangent segments must be displaced by the difference of the values of the failure criteria diameter $D_{fX}$ and the cone-shaft equivalent diameter $D_{eqB/P}$.

$$D_{\theta_a} = \frac{32 \cdot M_{tmax} \cdot (L - l_L - l_R)}{\pi \cdot G \cdot \theta_a}$$
Having determined the shape (through the tangent equations) and the maximum diameter \((D_{\text{max}} \text{ or } D_{\text{max,NEW}})\) regarding endurance, deformation and vibration failure criteria, it is now possible to calculate raw material volume \((V_M, \text{Equation 5})\) and the final material volume \((V_F, \text{Equation 6})\).

\[
V_M = L_0 \cdot \frac{\pi}{4} \left(2 \cdot D_{\text{max}} + D_{fX} - D_{eqX}\right)^2
\]  
(5)

\[
V_F = \pi \cdot \left[ \int_{x_{\text{max}}} (L - L_s)/2 \left[ D_L(x) + \frac{(D_{fX} - D_{eqX})}{2} \right]^2 dx + \int_{x_{\text{max}}} (L_o + 2L)/2 \left[ D_R(x) + \frac{(D_{fX} - D_{eqX})}{2} \right]^2 dx \right]
\]  
(6)

3.3 Shaft cost estimation

The shaft manufacturing costs consist of the material and the machining cost. Material costs depend on the raw material volume and the kind of material. Machining costs depend on the kind of the machining processes that the shaft will undergo. Common processes in this case are sawing, turning, boring, grinding, milling, forging (for large shafts), heat and surface treatment. Usually, material, heat and surface treatment costs are volume or weight dependent. Those direct costs \(C_{MD}\) are given in Equation (7). In this case, \(k^*\) is the relative material cost rate; \(k_{Vo}(V)\) is the cost rate of the reference material in €/m³ [17]; \(k_h\) and \(k_s\) stand for the cost rates for heat and surface treatment in €/m³; \(\rho\) is the material density in kg/m³.

\[
C_{MD} = \left(k^* \cdot k_{Vo}(V) + (k_h + k_s) \cdot \rho\right) V_M
\]  
(7)

The costs for the manufacturing processes depend on the process time. This time consists of the sum of the primary time \(t_p\) (which depend on the material volume to be processed), the secondary time \(t_s\) and the set up time \(t_s\) (both depend partially on the shaft length, volume and weight as well as on the machine and operation type). Typical values for \(t_n\) and \(t_s\) can be estimated either from company records or from the bibliography (see [12, 14]). Since now the shape of the shaft is already approximated, it is possible to calculate the primary times for turning, sewing or forging. Equation (8) gives exemplarily the primary time for turning \(t_{ptur}\) (\(a_p\) stands for the depth of cut in mm, \(f\) for the feed in mm/rotation and \(v_C\) for the cutting speed in mm/s). For milling, boring and grinding the process amount (length and cut depth or number of the cuts) must be estimated separately since they depend on the elements, which will be assembled on the shaft. Concerning the rest of the process parameters (i.e. feed, cut depth, etc.) typical values can be chosen.

\[
t_{ptur} = \frac{V_M - V_F}{a_p \cdot f \cdot v_C}
\]  
(8)

Since here the short differentiating job order costing method is used, the total material costs consist of the material direct costs (7) including the indirect costs (considered through an indirect cost unit rate \(g_{MGK}\)). The sum of the products of the process times and process cost rates represents the machining costs. Thus, the shaft manufacturing costs \(C_{Shaft}\) including “n” machining processes, material, heat and surface treatment are given by the Equation (9). The parameter \(c_m\) stands for the cost rate of the “i-th” manufacturing process in €/s or €/min and \(N\) for the manufacturing lot size.

\[
C_{Shaft} = \sum_{i=1}^{n} c_m \cdot t_{pi} + C_{MD} + \text{Indirect Costs}
\]  
(9)
\[ C_{Shaft} = C_{MD} \cdot (1 + g_{MGK}) + \sum_{i} \left[ c_{mi} \cdot \left( t_{pi} + t_{ni} + t_{si} / N \right) \right] \] 

(9)

3.4 Shaft cost estimation tool

Aiming to saving time, the method is implemented in a software module (Figure 5). In an appropriate structured interface, the user must choose a shaft topology and enter some rough geometry, load and design constraint data (safety and threshold values for endurance, deformation and vibration), which will be used to approximate the initial and the end shaft volume. Then, the shaft machining processes must be chosen and eventually some corresponding process data must be entered. Since cost and process data depend on company potential and analogue records, they can be considered to be constant for a period of time and so they can be saved for future estimations. Hence, the manufacturing costs of a shaft can be estimated. The assignment of error/range values to some input variables combined with the utilization of the Gauss formula, leads to the calculation of an error/range value for the estimation. This provides an idea for the estimation’s confidence range. Moreover, the manufacturing cost sensitivity regarding each parameter used for the calculation is provided. Hence, the way that individual design and processing variables affecting costs can be localized and so critical optimization parameters and objectives concerning the shaft manufacturing costs can be defined already in the concept phase.

Figure 5. Software tool for the method implementation.
4. Conclusions

A method for the estimation of the manufacturing costs for newly designed or prototype shafts in the early concept phase was presented. The method utilizes the design principle of uniform strength as well as endurance, deformation and vibration formulas in order to conduct the cost forecast based on PDS-data. Besides, the use of Gauss’s systematic error propagation formula, allows the calculation of an error-tolerance value for the cost estimation. The implementation of the method in a software module makes the prompt estimation possible while through the provided sensitivity analysis, cost optimization objectives can be defined. The prototypical application of the method showed that an estimation error under 10% can be achieved. This is important, because for the early development phases estimation errors (i.e. with parametric equations or functional relationships) of 20-30% are generally achieved [2, 3].

The method can be further implemented for shaft topologies beyond those presented here or for more “challenging” cases like large aggregate rotors (i.e. turbines). In this case, additionally critical parameters for each special application must be considered. For instance, in the case of gas turbine rotors an additional variable in Equation (1) could include the influence on the material strength of high temperatures as well as that of the shaft material hardening due to special treatment. Besides, an additional constraint for a free of vibration operation could be regarded. Here, the influence of the large number of stepped sections could be considered through a multiplication factor for the secondary and set-up time (therefore, the similarity of the studied shaft application with other in the past is significant), since the primary time depends on the material volume to be processed (forged or removed).

The sub-procedures of the method can be expanded also to other machine elements, such as beams, linkages or free shape structures, when geometric functions and material threshold equations using requirement list data can be generated in order to be combined with the formula resulting from the uniform strength principle. Thus, through the determination of the process volume and the manufacturing processes, a cost forecast can be achieved.

References

ASME commemorating the 50th anniversary of the Design Engineering Division of the ASME), 1995, pp. 25–32.


