Abstract
This work considers the coupling between finding an optimal approach for the design of a product and a subjective study to define desirable features. It relies on two domains which remain generally distinct: the design with a scientific approach (generally math-based) and the design with a sensory and perceptual approach (subjective). The paper describes a methodology for a user-centered design, applied to the design of musical instruments (trumpet). The quality of musical instruments as it is perceived by musicians was studied with sensory analysis, which seems as far as we know to be original on musical instruments. Two types of study were carried out on a set of trumpets: firstly, a sensory study, which aim is to characterize the perception of the instruments by musicians; secondly, an objective study, which consists in an objective description of the instruments by physical measurements. A paramount characteristic of the acoustic behavior of brasses, the acoustic input impedance, has been measured. We correlated the sensory profiling data and the instrumental measurements, in order to deduce useful objective functions for the design of new instruments. The design of a new instrument was finally made by multicriteria optimization of the objective functions, using genetic algorithms.

Keywords: Product design, user-centered design, optimal design, sensory analysis, genetic algorithm, musical acoustics.

1 Introduction
In today’s highly-competitive market, developing new products that meet consumers’ needs and tastes is a crucial issue. To be successful, a product should not only satisfy objective requirements, but should also satisfy the consumers’ tastes – inherently subjective. Improving the perceived quality and the craftsmanship of products is then an important challenge in product design. This objective is not simple to reach because it needs to include, in the design loop, a very complex entity: the human. We thus need to develop specific design methodologies which can take into account customers’ feelings and preferences during the design process [1] [2]. Such methodologies mainly use practices developed by the food industry (e.g. sensory analysis), which are now used for design by being progressively applied to all senses which affect perception. In the same way, the Kansei Engineering [3], developed by Japanese researchers, aims to investigate customer feeling and proposes an ergonomic, consumer-oriented technology for product design.

Our work lies in this context. We propose in this article to show how the perceptions of users can be taken into account in order to improve the product design. We developed a user-centered methodology in order to optimize certain perceived attributes of a product. To
describe our approach, we focused on a particular product for which the perceived aspects play a very important role in the assessment of the quality: a brass musical instrument (trumpet). Several objective and subjective studies were carried out by the past on musical instruments [4] [5] [6], but few of them tackles the coupling between these two approaches.

The final objective of this study is to provide brass-instrument makers with useful tools to better know musicians’ desires and to have efficient techniques to satisfy them. From a design research point of view, the objective is to develop a generic design methodology which can be applied to every kind of product for which the aesthetical or emotional aspects are preponderant.

We present in section 2 the user-centered methodology we developed, based on sensory analysis techniques and optimization procedures. Section 3 is dedicated to the perceptual studies of a set of trumpets, using a panel of musician-experts. The objective study of the instruments (measurement of the acoustic impedance) is presented in section 4. In section 5, data analysis techniques are used to study the correlations between the subjective and the objective data. Section 6 tackles the design of a new instrument by optimization procedure (genetic algorithms). Conclusions and perspectives are drawn in section 6.

2 Description of the user-centered methodology

The proposed methodology is inspired by many methods and tools developed for the design of products in various domains: food industry, automobile industry, industrial design, kansei engineering, psychoacoustics, etc.

It is decomposed into several stages:

1. Set up of a product space, made of existing products which roughly all answer the same usage functions, but differ according to their performances, style, aesthetics etc. The chosen products must be different enough in order to stimulate a wide sensory range of the user, but similar enough in order to remain in the same sensory domain [7].

2. Perceptual analysis of the product space. This stage uses sensory analysis tools (attributes, panel of experts, sensory profiling). After a training period for the rating of the attributes, the assessors perform the sensory profile of the products.

3. Objective analysis of the product space. This consists of measuring various objective physical characteristics of the product, and, after a physical analysis of the product, to propose objective criteria which condition the perceived sensations.

4. Study of correlations between the sensory attributes and the physical characteristics.

5. Definition of the need. The need corresponding to a new product is specified according to the sensory attributes. Various techniques based on preference-mapping can be used to detect customers’ preferences.

6. Definition of the technical specifications. The correlations are used to translate the requirements into technical specifications according to the physical characteristics.

7. Optimization: Formulation of the design problem as an optimization problem:
   a. Definition of the objective functions and the constraints,
   b. Definition of the optimization variables
   c. Choice of a optimization strategy – determination of optimal solutions
8. Manufacturing of the “optimal” products and test.
The synoptic of the methodology is described figure 1.

We propose to describe each stage of the methodology on a particular example: brass musical instruments (trumpets).

3 Perceptual analysis of musical instruments (trumpets)

3.1 Background: functioning of brass musical instruments

Brass wind instruments (trumpet, trombone etc.) are musical instruments for which an input pressure $P_a(t)$ and an air flow produced by the musician generate oscillations of a mechanical device (the lips of the musician) and create a variable pressure $P_{in}(t)$ in the mouthpiece (Figure 2). These variations of the acoustic pressure are next propagated outside the instrument by the way of the bell, and produce an external sound $P_{ext}(t)$. The sound produced is the result of a complex coupling between the excitator (the lips) and the resonator (the instrument): the mechanical characteristics of the excitator and the acoustical characteristics of the resonator have both an influence on the sound produced.

Several notes can be played by modifying the mechanical characteristics of the lips (the “embouchure” of the musician), and/or by changing the geometry of the resonator (use of a slide for the trombone, or valves for the trumpet). The main design variables of the instrument, which condition the perceived quality by the musician, are:

- The dimensions of the internal geometry of the resonator, called “the bore”. The acoustic behavior of the resonator is strongly dependant of the inner form of the resonator.
• The surface roughness, which generates viscothermal loses.
• The quality and stiffness of the construction.
• The type of material and the forming process; these can have a perceptible influence on the vibrations of the wall.
• The internal gaps between the parts, which affect the air-tightness of the resonator.

From these variables we have chosen to study the influence of the internal geometry of the resonator on the perceived quality of the instrument.

3.2 Setting up of the product space

In order to design a set of trumpets which are very different in playing condition, we decided to parameterize the shape of a very influential part of the resonator on the acoustic behavior of the instrument: the leadpipe. This part is roughly conical and is located between the mouthpiece and the tuning slide (Figure 2).

From the measurements of the internal form of existing leadpipes (measured with calipers), we designed a new leadpipe made of 4 different interchangeable parts, each conical and parameterized by the radii $r_1, r_2, r_3, r_4$ (Figure 3).

![Figure 3: design of the parameterized leadpipe](image)

Several parts 1-2-3-4, with various values for the radii $r_1, r_2, r_3, r_4$, have been manufactured with a numerically controlled turning machine. The proposed values of $r_1, r_2, r_3, r_4$ correspond roughly to dimensions of marketed leadpipes, and the assembling of the parts allows the generation of various inner profile of leadpipes (many hundreds). A coding of each leadpipe, made of 4 letters (one letter for each part, the letter corresponding to a given dimension of the radius), has been defined in order to distinguish the leadpipes.

So, using the same trumpet (*Bach* model *Vernon*, bell 43) and the parameterized leadpipe, several hundred of different instruments with notably different acoustical behavior can be designed. With this device, we finely control the variation of the design parameter of the set of instruments. Furthermore, the musician is not able to recognize which leadpipe he/she tests. This will be a very important property in order to check the repeatability of the musicians’ assessments.

3.3 Training and assessment of the panel of experts

A panel of 10 professional trumpet players has been set up for the perceptual analysis of the instruments.

Before the assessment of the instruments, we first worked on the definition of relevant terms to describe their quality. We used an approach based on sensory analysis and the sensory profiling: the musicians were involved in a group session and a free-verbalization task on instruments of various quality. We defined a set of sensory attributes, characteristic of the quality of a trumpet, and proposed evaluation procedures for each of these attributes (table 1).
Table 1: list of the sensory attributes characteristic of the quality of trumpets

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Definition</th>
<th>Range</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intonation</td>
<td>Relative position of the notes</td>
<td>out of tune / in tune</td>
<td>arpeggio</td>
</tr>
<tr>
<td>Test note E</td>
<td>Difference of height note E (fingering 0) and note E (fingering 12)</td>
<td>similar / different</td>
<td>play notes E(0)-E(12)</td>
</tr>
<tr>
<td>Centering</td>
<td>Ability of the instrument to be centered on a note</td>
<td>bad / good</td>
<td>attack of the note G4</td>
</tr>
<tr>
<td>Response</td>
<td>Ability of the instrument to play immediately</td>
<td>bad / good</td>
<td>Detached notes</td>
</tr>
<tr>
<td>Low register</td>
<td>Width of the Dynamic range</td>
<td>limited / big</td>
<td>dynamics pp, mf, ff</td>
</tr>
<tr>
<td>Medium register</td>
<td>Width of the Dynamic range</td>
<td>limited / big</td>
<td>dynamics pp, mf, ff</td>
</tr>
<tr>
<td>High register</td>
<td>Width of the Dynamic range</td>
<td>limited / big</td>
<td>dynamics pp, mf, ff</td>
</tr>
<tr>
<td>Timbre</td>
<td>Tone of the instrument</td>
<td>dark / bright</td>
<td>comparison/reference</td>
</tr>
</tbody>
</table>

A training session of the panel of experts has been conducted with a set of 4 instruments (4 different leadpipes). Each musician was asked to rate the trumpets according to the attributes of table 1 on a no-structured scale. In order to evaluate the repeatability of the experts, 3 replications of the same instrument was provided, the order of presentation of the instruments being randomized.

A two-way analysis of variance with interaction was employed to study the effect of the product, the effect of the expert and the interaction between product and expert on the assessments of the attributes. The value of the “statistics” F has been compared to the threshold value of the Fisher-Snedecor table with p-value = 0.05. The factors for which the effect is significant are shaded in table 2.

Table 2: results of the two-way analysis of variance.

<table>
<thead>
<tr>
<th></th>
<th>Intonation</th>
<th>Test E</th>
<th>Centering</th>
<th>Response</th>
<th>Low register</th>
<th>Medium register</th>
<th>High register</th>
<th>Timbre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expert effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interaction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The “product effect” is significant except for the attributes “centering”, “response” and “low register”. For these attributes, further training sessions are needed to improve the accuracy and the repeatability of the experts, and/or to improve the assessment procedures.

The “expert effect” is significant for all the attributes. This signifies that the experts assess differently the instruments, because they don’t define the scale in the same way. This is a classical characteristics in sensory analysis and is not an obstacle to exploit the results.

The interaction “product*expert” is significant for only the attribute “intonation”. This interaction causes a problem in interpreting the results because it signifies that the agreement between the experts is bad. An average value of the experts’ scores could be in this case not representative. By studying the raw data of the evaluations, we found out that one expert proposed opposite evaluations relatively to the rest of the group. For this expert, further training sessions are needed to explain the attribute “intonation”.

Given that we obtained very promising results with this training session, we decided to use the panel of experts for an assessment task. The product space consisted of a set of 12
different instruments, defined by their leadpipe code (table 3). Two replications have been
proposed, the order of presentation of the trumpets being randomized.

The experts were asked to give the sensory profile of each instrument. In this article, we
propose to exploit the assessments relative to the attribute “intonation”. The average
subjective score of intonation for each leadpipe is presented in table 3.

4 Objective analysis of musical instruments

4.1 The input impedance $Z_{in}$

Brass wind instruments (and, more generally, wind instruments) can be characterized by their
acoustic impedance $Z_{in}$, the transfer function between the acoustic flow $Ue$ and the acoustic
pressure $Pe$, which depends on the frequency $\omega$ (equation 1):

$$Z_{in}(j\omega) = \frac{Pe(j\omega)}{Ue(j\omega)}$$  \hspace{1cm} (1)

This quantity can be calculated or measured [8]. It’s a very important property for the
characterization of a brass instrument: it gives the magnitude of the acoustic response to a
forced oscillation. The typical input impedance of a trumpet presents several peaks of
impedance, called the partials of the resonator (figure 4).

![Figure 4: Input impedance of a trumpet (magnitude)](image)

In playing situation, the musician produces a note whose frequency (the playing frequency) is
close to the resonance frequency of an impedance peak [9]. In first approximation, the playing
frequency (which conditions the intonation) is mainly governed by the corresponding peak of
the impedance1.

4.2 Objective variables extracted from the impedance $Z_{in}$

The input impedance of the 12 trumpets proposed for the subjective evaluation has been
measured with the BIAS device [10]. The frequencies of the peaks, from partial n°2 to partial
n°10, are given in table 3.

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1 The study of sound production in brass shows that there is a complex aeroelastic coupling between the lips of
the musician and the resonator. Thus, the intonation of the instrument is not only controlled by the closest
resonance frequency but possibly conditioned by upper resonance frequencies of the resonator.
Table 3: resonance frequencies of the peaks and score of intonation of the 12 trumpets

<table>
<thead>
<tr>
<th>Code</th>
<th>$f_{\text{max}(2)}$</th>
<th>$f_{\text{max}(3)}$</th>
<th>$f_{\text{max}(4)}$</th>
<th>$f_{\text{max}(5)}$</th>
<th>$f_{\text{max}(6)}$</th>
<th>$f_{\text{max}(7)}$</th>
<th>$f_{\text{max}(8)}$</th>
<th>$f_{\text{max}(9)}$</th>
<th>$f_{\text{max}(10)}$</th>
<th>Subjective score of intonation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABFN</td>
<td>230.5</td>
<td>343.5</td>
<td>454.5</td>
<td>574</td>
<td>691</td>
<td>801.5</td>
<td>901.5</td>
<td>1019</td>
<td>1144</td>
<td>7.2</td>
</tr>
<tr>
<td>ACHN</td>
<td>230</td>
<td>344.5</td>
<td>457</td>
<td>575.5</td>
<td>690</td>
<td>801</td>
<td>904.5</td>
<td>1022</td>
<td>1144</td>
<td>8.8</td>
</tr>
<tr>
<td>ADKN</td>
<td>229.5</td>
<td>345.5</td>
<td>460</td>
<td>576.5</td>
<td>687.5</td>
<td>796.5</td>
<td>906</td>
<td>1024</td>
<td>1143</td>
<td>9.5</td>
</tr>
<tr>
<td>BFLN</td>
<td>229.5</td>
<td>346</td>
<td>460.5</td>
<td>577.5</td>
<td>687.5</td>
<td>796.5</td>
<td>907</td>
<td>1025.5</td>
<td>1144</td>
<td>8.9</td>
</tr>
<tr>
<td>BFOS</td>
<td>229</td>
<td>347</td>
<td>463</td>
<td>575</td>
<td>685</td>
<td>799.5</td>
<td>908.5</td>
<td>1021.5</td>
<td>1140</td>
<td>8.6</td>
</tr>
<tr>
<td>CGJQ</td>
<td>229.5</td>
<td>345.5</td>
<td>460</td>
<td>577.5</td>
<td>690</td>
<td>799</td>
<td>903</td>
<td>1021</td>
<td>1145</td>
<td>8.2</td>
</tr>
<tr>
<td>CHMQ</td>
<td>228.5</td>
<td>346</td>
<td>463</td>
<td>579</td>
<td>687.5</td>
<td>796</td>
<td>905</td>
<td>1023</td>
<td>1144</td>
<td>7.5</td>
</tr>
<tr>
<td>CHNR</td>
<td>229</td>
<td>347.5</td>
<td>465</td>
<td>580.5</td>
<td>689</td>
<td>798.5</td>
<td>908</td>
<td>1025.5</td>
<td>1147.5</td>
<td>6.6</td>
</tr>
<tr>
<td>CIPQ</td>
<td>228.5</td>
<td>346.5</td>
<td>464.5</td>
<td>581</td>
<td>687</td>
<td>796</td>
<td>907</td>
<td>1025.5</td>
<td>1146.5</td>
<td>6.1</td>
</tr>
<tr>
<td>DKLN</td>
<td>227.5</td>
<td>344.5</td>
<td>463.5</td>
<td>582.5</td>
<td>690</td>
<td>791.5</td>
<td>901.5</td>
<td>1025.5</td>
<td>1150</td>
<td>7.7</td>
</tr>
<tr>
<td>DKNR</td>
<td>227.5</td>
<td>345.5</td>
<td>465</td>
<td>581.5</td>
<td>688.5</td>
<td>794</td>
<td>902</td>
<td>1022.5</td>
<td>1146.5</td>
<td>5.1</td>
</tr>
<tr>
<td>DKOS</td>
<td>228</td>
<td>346.5</td>
<td>465.5</td>
<td>581.5</td>
<td>688.5</td>
<td>795.5</td>
<td>903.5</td>
<td>1022.5</td>
<td>1146.5</td>
<td>6.9</td>
</tr>
</tbody>
</table>

These objective data are next used to interpret the score of intonation.

5 Correlations

In this section, the correlations between the scores of intonation (subjective data) and the variables extracted from the impedance curve (objective data) are studied. The aim is to find out relations between these data in order to use them for the design of a new instrument. This approach needs some assumptions.

Let’s assume first that the global subjective assessment of intonation is a function of the intonation of the 5 main musical intervals in the tessitura of the instrument (2 octaves, 2 fifths, 1 third). Next, the hypothesis that is proposed to test is the following: “the intonation of these intervals depends on the following frequency ratios: $\frac{f_{\text{max}(3)}}{f_{\text{max}(2)}}$ and $\frac{f_{\text{max}(6)}}{f_{\text{max}(4)}}$ for the 2 fifths; $\frac{f_{\text{max}(4)}}{f_{\text{max}(2)}}$ and $\frac{f_{\text{max}(8)}}{f_{\text{max}(4)}}$ for the 2 octaves; $\frac{f_{\text{max}(5)}}{f_{\text{max}(4)}}$ for the third.

We propose to study the correlation between the intonation scores and the 5 explanatory variables $f_{\text{max}(3)} / f_{\text{max}(2)} - f_{\text{max}(6)} / f_{\text{max}(4)} - f_{\text{max}(4)} / f_{\text{max}(2)} - f_{\text{max}(8)} / f_{\text{max}(4)} - f_{\text{max}(5)} / f_{\text{max}(4)}$.

In other words, we want to test if a linear relationship between the intonation scores and the explanatory variables (predictors) can be proposed. Given that the explanatory variables are certainly correlated, a reduction of the dimensionality of the predictors via principal component analysis is necessary [11].

5.1 Representation by principal component analysis (PCA)

A normalized principal component analysis of the $p = 12$ individuals (trumpets) and the $n = 5$ variables (predictors) leads to the factorial plane plotted figure 5. More than 98% of variance is taken into account by only two factors $F_1$ and $F_2$: the initial data are effectively highly correlated.
5.2 Proposition of a model for the intonation

A quadratic model is proposed to interpret the scores of intonation $I_i$ by objective variables. Given that the predictors are correlated (see section 5.1), a regularized regression [11] is proposed (equation 2):

$$I_i = a.F_{1i} + b.F_{2i} + c.(F_{1i}^2 + F_{2i}^2) + d$$

The regression coefficients $a$-$b$-$c$, the determination coefficient $R^2$ and the statistics $F$ of the regression (table 4), are determined by minimization of the sum of the deviations squared.

Table 4: coefficients of the regularized regression

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>$R^2$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.03</td>
<td>2.6</td>
<td>-2.02</td>
<td>0.70</td>
<td>6.14</td>
</tr>
</tbody>
</table>

The adjustment of the data on the model is correct ($R^2 = 0.7$). Furthermore, the regression is significant with p-value=5% (Fisher Snedecor test: $F > F_{5\%}(3,8) = 4.07$).

The proposed model being an “ideal point model”, the extremum of the paraboloid (equation 2) can be plotted in the factorial plane (point Target, coordinates (Target$_{F1} = -a/2c$ ; Target$_{F2} = -b/2c$)) (figure 5). This extremum is a maximum of intonation, which indicates that, according to the data and our experts, the optimum of intonation would be located at this position of the factorial plane.

These results lead to the following comments:

- The assumption according to which the ratios of the resonance frequency are relevant for explaining the score of intonation seems to be confirmed ($R^2 = 0.7$ - significant regression),
- The assumption according to which the global assessment of intonation by the experts is a function of the intonation of 5 main musical intervals of the tessitura of the instrument seems to be confirmed too.
Next, with the coordinates of the target on the factors \( \text{Target}_F \) and \( \text{Target}_F \) as input, possible values of the initial variables are computed. This is done by using the coordinate transformation relation of the PCA (equation 3):

\[
F = XU
\]  

\( F \): matrix \((n \times p)\) of the factorial scores
\( X \): matrix \((n \times p)\) of the standardized initial data, generic term \( x_{ij} \)
\( U \): matrix \((p \times p)\) of the eigenvectors, generic term \( u_{ij} \)

With the sensible assumption that the value of the factorial scores of the Target on the principal component 3, 4, 5 is null, a unique solution is computed by solving the following linear system (equation 4):

\[
\begin{align*}
\text{Target}_F^1 &= u_{11}x_{11} + u_{21}x_{12} + u_{31}x_{13} + u_{41}x_{14} + u_{51}x_{15} \\
\text{Target}_F^2 &= u_{12}x_{21} + u_{22}x_{22} + u_{32}x_{23} + u_{42}x_{24} + u_{52}x_{25} \\
0 &= u_{33}x_{31} + u_{33}x_{32} + u_{33}x_{33} + u_{33}x_{34} + u_{33}x_{35} \\
0 &= u_{44}x_{41} + u_{44}x_{42} + u_{44}x_{43} + u_{44}x_{44} + u_{44}x_{45} \\
0 &= u_{55}x_{51} + u_{55}x_{52} + u_{55}x_{53} + u_{55}x_{54} + u_{55}x_{55}
\end{align*}
\]  

The initial data corresponding to this solution are given in table 5.

Table 5: ratios of frequencies corresponding to the Target, specifications for the design

<table>
<thead>
<tr>
<th></th>
<th>( \frac{f_{\text{max}}(3)}{f_{\text{max}}(2)} )</th>
<th>( \frac{f_{\text{max}}(4)}{f_{\text{max}}(2)} )</th>
<th>( \frac{f_{\text{max}}(5)}{f_{\text{max}}(4)} )</th>
<th>( \frac{f_{\text{max}}(6)}{f_{\text{max}}(4)} )</th>
<th>( \frac{f_{\text{max}}(8)}{f_{\text{max}}(4)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target</td>
<td>1.52</td>
<td>2.03</td>
<td>1.24</td>
<td>1.49</td>
<td>1.98</td>
</tr>
</tbody>
</table>

We notice that the ratios of frequencies of this instrument are almost harmonic. This property was not at all obvious before the study. Indeed, the intonation is a subjective characteristic, and it’s not a priori proved that harmonic ratios are more desirable than others. For example, piano tuners are quite aware of this fact and never use an electronic tuner to do their task. Only an approach like the one proposed in this article is able to define specifications by taking into account the feelings and perceptions of users.

With this study on the attribute « intonation » of a trumpet, we defined the specifications of a new instrument. The next step is to design such instrument by optimization techniques. More precisely, our objective is to find the inner form of the leadpipe which would have the characteristics of the Target (specifications given table 5).

6 Optimization

6.1 Introduction

A lot of optimization methods are proposed and used in design, like calculus-based (gradient-based), enumerative or heuristics methods [12]. The two first schemes can be subjected to a lack of robustness if the objective function is not defined, not continuous or not derivable and a lack of efficiency in the case of very large design spaces. Concerning the design of brass
instruments, a mono objective optimization using the Rosenbrock algorithm is for example proposed in [13].

We have been interested in a multi-objective optimization of brasses, and we have chosen to use a search procedure based on random choices, which doesn’t necessitate the calculation of the gradient : the genetic algorithms [14]. This stochastic optimization algorithm provides generally a family of “good” solutions in an acceptable calculation time, and is for many fields of applications an interesting alternative to gradient-based optimization [14]. It enables furthermore an exploration of a large design space.

6.2 Genetic algorithms

The principle of the progressive genetic algorithm used for our application is described in figure 6.

A population is composed of N individuals. The choice of the size of the population is the result of a compromise between a reasonable calculation time and a wide covering of the design space. Generally, N is chosen to be around 20 or 30 times the number of design variables. To each individual of the initial population, selected at random, is associated an attribute called “fitness”. Fitness is the rate of adaptability of an individual to the environment, like the ability to survive in the Darwin theory. Higher the fitness, bigger the chance to be in the next generation.

With the value of each individual according to the objective function as input, the population can be sorted easily. In the case of multi-objective functions, the dominance relation which allows the definition of the rank of the individuals and of the Pareto set, is applied:
• Dominance relation: to dominate another individual, an individual must be at least as good on all the objectives and better on at least one objective. The rank of an individual is equal to the number of times it is dominated +1,

• Pareto set: set of the not-dominated individuals (rank 1).

Generations of new populations are made until the convergence criteria is reached (maximum number of generations –or- threshold of acceptability –or- all individuals of rank 1).

The generation of a new population is composed of 5 stages (figure 7):

• Selection of 2 parents ; randomly or according to their performance,
• Crossing over or reproduction of the parents in 2 children,
• Mutation (or not, according to a probability),
• Calculation of the fitness of the children and their rank,
• Selection and insertion in the pop. of the 2 best among the 2 parents and 2 children.

To make possible a complete replacement of the population, these 5 stages are repeated a number of times equal to the half of the population size (N/2).

![Figure 7: description of the reproduction, mutation and cross-over procedures](image)

In the process of generation, the differences between various strategies lie in the first and the fifth stage. Indeed, progressive genetic algorithms allow ones to specify the selection of parents and replacement method (totally at random, against the worst, against the most similar…) [15]. Our choices are presented in the next paragraph.

6.3 Calculation of $Z_{in}$: the transmission line model

The calculation of the input impedance has been made by a theoretical approach based on the transmission line modeling [8].

The instrument is modeled as the juxtaposition of cylindrical and conical segments, defined by their geometrical data (length, input and output diameters) (Figure 8). Three design variables, the inner radii $r_2$, $r_3$, $r_4$ of three particular sections of our homemade leadpipe, have been defined for the optimization problem. The first radius $r_1$ was fixed, in order to join continuously with the previous part of the leadpipe, i.e the mouthpiece.
The relation between the acoustic pressure $P_i$ and volume flow velocity $U_i$ at the input and output of an element $i$ is given by equation 5, where $H_i$ is the transmission matrix for element $i$:

$$
\begin{bmatrix}
P_i \\
U_i
\end{bmatrix} = 
\begin{bmatrix} H_{i1} & H_{i2} \\
H_{i21} & H_{i22}
\end{bmatrix}
\begin{bmatrix}
P_{i+1} \\
U_{i+1}
\end{bmatrix} = H_i
\begin{bmatrix}
P_{i+1} \\
U_{i+1}
\end{bmatrix}
$$

(5)

For cylindrical and conical segments, the expression of $H_i$ according to the geometry can be found in [8]. The transmission matrix $H$ of a resonator consisting of $N$ segments is the product of the individual transmission matrices (equation 6):

$$
H = \prod_{i=1}^{N} H_i
$$

(6)

The input impedance $Z_{in}$ of the resonator is finally given by equation 7:

$$
Z_{in} = \frac{P_{in}}{U_{in}} = \frac{H_{12} + H_{11}Z_L}{H_{22} + H_{21}Z_L}
$$

(7)

where $Z_L$ is the radiation impedance (termination load impedance of the waveguide). The simplest model is to suppose that $Z_L$ is equal to 0, but more sensible assumptions can be proposed to determine $Z_L$ [8].

Using calipers, the bore of the trumpet used for the tests has been measured. The input impedance of a “current” trumpet (with fixed values of the design variables $x = [r_2, r_3, r_4]$) can then be calculated. The peaks are extracted from the impedance curve, and put in ratio corresponding to the values: $f_{max}(3) / f_{max}(2) - f_{max}(6) / f_{max}(4) - f_{max}(4) / f_{max}(2) - f_{max}(8) / f_{max}(4) - f_{max}(5) / f_{max}(4)$.

The design problem is finally translated into the following multicriteria optimization problem (equation 8):

$$
\begin{align*}
\text{minimize} & \quad e_1 = \frac{f_{max}(3)}{f_{max}(2)} - \frac{f_{max}(3)}{f_{max}(2)} \\
& \quad e_2 = \frac{f_{max}(6)}{f_{max}(4)} - \frac{f_{max}(6)}{f_{max}(4)} \\
& \quad e_3 = \frac{f_{max}(4)}{f_{max}(2)} - \frac{f_{max}(4)}{f_{max}(2)} \\
& \quad e_4 = \frac{f_{max}(8)}{f_{max}(4)} - \frac{f_{max}(8)}{f_{max}(4)} \\
& \quad e_5 = \frac{f_{max}(5)}{f_{max}(4)} - \frac{f_{max}(5)}{f_{max}(4)}
\end{align*}
$$

(8)
The objective functions are the deviation between the target values (table 5) and the current ratios \( \frac{f_{\text{max}}(3)}{f_{\text{max}}(2)} - \frac{f_{\text{max}}(6)}{f_{\text{max}}(4)} - \frac{f_{\text{max}}(4)}{f_{\text{max}}(2)} - \frac{f_{\text{max}}(8)}{f_{\text{max}}(4)} - \frac{f_{\text{max}}(5)}{f_{\text{max}}(4)} \) (extracted from the calculation of \( Z_{in} \)). The design problem is finally to determine the values of \( x = [r_2, r_3, r_4] \) which minimize the 5 objective functions.

### 6.4 Implementation

Genetic algorithms have been used to solve the multicriteria optimization problem (equation 8). The design variables \( x = [r_2, r_3, r_4] \) are digitized in the interval \([0.002;0.01]\) by step of \(5 \times 10^{-5}\). This generates 160 possible values, thus coded in binary on 8 bits. Let notice that a systematic exploration of all the design space would lead in this simple case to an unacceptable computation time\(^2\). A chromosome, or individual, characterized by a value for \( x = [r_2, r_3, r_4] \), represents the inner geometry of a leadpipe, and corresponds to a trumpet. A chromosome is thus coded on 24 bits. The initial population is made of \( N = 60 \) trumpets.

Concerning the convergence criteria, a threshold of acceptability (equal to 0.01) has been defined for the 5 objective functions \( e_i \). If \( e_i < 0.01 \ \forall \ i \), the convergence criteria is satisfied and the algorithm stops. Six control parameters of the algorithm can be adjusted: the maximum number of generations, the population size, the probability of mutation, the probability of cross-over, the selection method of the parents and the selection method between mates and children. Table 6 recap the control parameters used for the application.

<table>
<thead>
<tr>
<th>Number of generations</th>
<th>N (pop size)</th>
<th>Probability Of mutation</th>
<th>Probability of cross-over</th>
<th>Selection of the parents</th>
<th>Selection between mates and children</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>60</td>
<td>0.15</td>
<td>0.8</td>
<td>Randomly</td>
<td>Replace by the best</td>
</tr>
</tbody>
</table>

### 6.5 Applications – Results

The results of the multicriteria optimisation is given table 7. Five not-dominated solutions \( S_i \) (rank 1) were extracted of the Pareto set. This is done by considering an a priori preference of the decision maker, such as the sum of the deviations for all the objectives is minimum (equation 9):

\[
\text{Sum}(j) = \sum_{j=1}^{5} e_i
\]

Table 7: descriptions of five solutions of the Pareto set

| \( S_1 \) | \( e_1 \) | \( e_2 \) | \( e_3 \) | \( e_4 \) | \( e_5 \) | \( \text{Sum}(j) \) | \( r_1 \) (mm) | \( r_2 \) (mm) | \( r_3 \) (mm) |
|-----------|---------|---------|---------|---------|---------|--------------------|...............|...............|...............|
| 0.0159    | 0.0933  | 0.0376  | 0.0861  | 0.0272  | 0.2601  | 5                  | 9.8          | 9.8          | 9.8          |
| 0.0162    | 0.0938  | 0.0377  | 0.0853  | 0.0272  | 0.2602  | 7.1                | 6.5          | 4.4          | 4.4          |
| 0.0162    | 0.0938  | 0.0377  | 0.0853  | 0.0272  | 0.2602  | 7.1                | 6.4          | 4.4          | 4.4          |
| 0.0159    | 0.0933  | 0.0377  | 0.0863  | 0.0272  | 0.2604  | 7.5                | 7.7          | 4.5          | 4.5          |
| 0.0162    | 0.0938  | 0.0377  | 0.0855  | 0.0272  | 0.2604  | 9.1                | 8            | 4.4          | 4.4          |
| DKOS      | 0.0177  | 0.0958  | 0.0377  | 0.0865  | 0.0272  | 0.2649             | 5.5          | 5.5          | 6            |

\(^2\) To calculate all the possible solutions, with three design variables coded on 160 values, around one and a half month of calculation would be necessary on a Personal Computer.
According to the calculation of the objective functions, the “best” leadpipe among the twelve used for the tests is DKOS (“best” according to the single criteria Sum(j)). The performances of this leadpipe are given for information in table 7. Of course, and fortunately, the performances of the optimized leadpipes are better than those of DKOS.

The leadpipe S1 (r2 = 5mm ; r3 = 9.8mm ; r4 = 9.8mm) is finally the best leadpipe according to the criteria Sum(j). The continuation of this work will be to manufacture it and to test it with the panel of experts, in order to validate the proposed methodology.

7 Conclusions

We presented in this paper a methodology for a user-centered design. It was applied to the design of brass musical instruments (trumpets), and on a particular attribute of the perceived quality of trumpets: the intonation. All the stages of the methodology were clearly described on this particular example. After a subjective study using a panel of experts-musician, the specifications of a new instrument have been defined. The design of this new instrument was done by multicriteria optimization using genetic algorithms.

Two kinds of results are provided by the study. Firstly, concerning musical acoustics, this work is an original approach to study the perceived quality of musical instruments. Using a consensus vocabulary and assessment procedures, several sensorial attributes have been defined by the assessors. Correlations between sensorial attributes and acoustics measurements have been scientifically studied, and they provide interesting links between the subjective and the objective world.

Secondly, concerning the design methodology, our study proposes an integrated approach which puts the user in the center of the design loop. The approach starts with the user and his/her perception, proposes the definition of an objective function coherent with the user’s feeling, and finally provide an optimal solution which can be tested. It’s a proposition to show how an optimization approach (convergent thinking) and a subjective and emotional study with sensory analysis (divergent thinking) can be combined. It is a first step for a more rational treatment of the subjective aspects of the need in product design.

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