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USING SIMILARITY RATIOS FOR FINDING, EVALUATING AND OPTIMIZING PRINCIPLE SOLUTIONS

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1. Introduction

Finding principle solutions accomplishing tasks is a main goal of methodical engineering design. It is often difficult to find suitable structures from the wide range of domain specific partial solutions which can be combined compatibly according to a required functional interrelationship. Furthermore, the solutions should contain the potential for quantitative optimization. The solutions have to fulfil complex holistic requirements in general, e.g. low product life cycle costs, a required dynamic behaviour or low noise emission. The solution space is restricted, for instance because of a fixed design space. For an evaluation of the fulfilment of requirements, an early approximate calculation ist always helpful and necessary to some extent. To find solutions efficiently during the conceptual design phase, a continuous usable product-representing model [Franke1976] would be desirable. A suitable product-representing model has to support the synthesis of solutions [Chakrabarti2002], should comprehend all relevant solution parameters and it enables quantitative evaluation. The paper points out that nondimensional similarity ratios (e.g. [Buckingham1914], [Baker1991]) can be applied conveniently.

Similarity ratios are characteristic power products of measurable variables, like geometric dimensions, forces and costs, which can be described by a measured value and a dimension unit. An important basis for establishing dimensionless ratios is the Pi theorem [Buckingham1914]. It states that each dimensional homogeneous relation $f(x_1,...,x_n) = 0$ between n physical variables x_i can be represented completely by a reduced number of m dimensionless, linear independent quantities, the so-called π-ratios. The π-ratios can be identified by a list of the relevant physical parameters $(x_1,...,x_n)$. The number of π-quantities corresponds to the rank r of the dimensional matrix. The quantities can be calculated by using the following equation:

$$\pi_j = x_j \cdot \prod_{i=1}^{r} x_i^{-\alpha_{ij}}, \; i \in \{1,..., r\}, \; j \in \{1,..., m\}, \; \alpha_{ij} \in \mathbb{R}, \; m = n - r \qquad (1)$$

Figure 1 clarifies the calculation of a complete set of π-ratios considering as example the deformation of a pressure spring which is loaded by a force. The reduction of parameters simplifies the representation of solutions remarkably. The essential design influences can be identified, comprehended and used much more easily. Considering the conceptual design of an electrically switched friction clutch as an example, a methodical procedure based on similarity ratios is demonstrated.

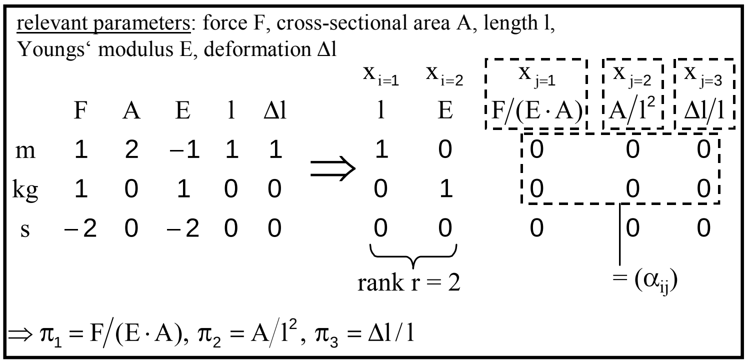


Figure 1. Establishing dimensionless ratios with the aid of the Pi theorem

2. Searching for complete principle solutions

To reduce the complexity during the search for solutions, the overall function has to be broken down into less complex sub-functions first. Afterwards partial solutions have to be found for these sub-functions. The partial solutions have to be combined compatibly according to the functional interrelationship. Figure 2a shows the function structure of a friction clutch. The graphical symbol (1) represents the sub-function of an electromechanic energy conversion, for which a broad search for solutions should be realized in this case study.

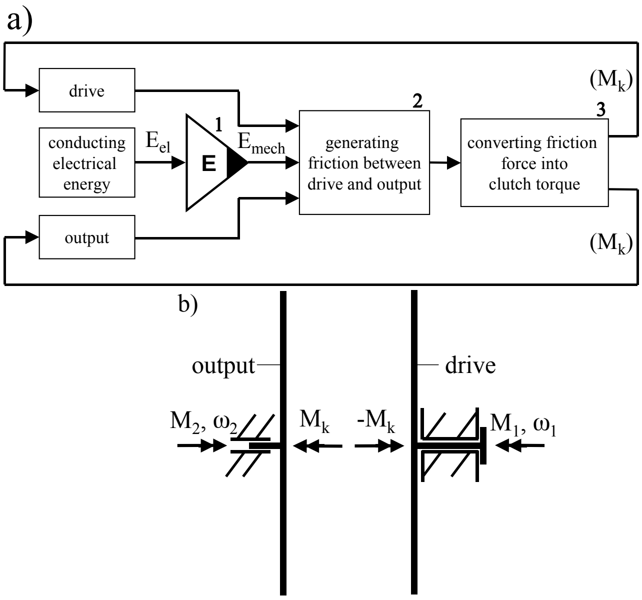


Figure 2. Friction clutch: a) function structure, b) arrangement drawing

2.1 Selection and classification of partial solutions based on physical quantities

Based on contributions to the universal system theory, e.g. in [Macfarlane1964], the author developed a systematic approach to evaluate the suitability of solutions to fulfil design functions at an early stage of the design process.

The approach derives the suitability from the physical quantities, like force or velocity, contained in the quantitative descriptions of the solutions. If the relevant parameters are available, e.g. formalized as dimensionless ratios, a methodical selection and classification of solutions can be realized by using logical and set theoretical relations of the characteristic parameters (see figure 4).

The processes of energy transformation in technical systems, like the conduction and storage of energy, can be described completely by "intensity" and "quantity" parameters and the effort and flow storage variables (T and P) well-known from system dynamics [Macfarlane1964].

a)

			mechanical		electro-magnetical	fluidic	thermo-dynamic
			trans-lation	rotation			
			k = 1	k = 2	k = 3	k = 4	k = 5
flow variable	intensity variable	τ_k	F	M	I	dV/dt	dS/dt
	quantity variable	σ_k	p_i	L	Q	V	S
effort variable	intensity variable	α_k	v	ω	U	p	ϑ
	quantity variable	λ_k	s	φ	Φ	$\int p\,dt$	--
T-storage		T_k	c	c_φ	L_m	M_F	--
P-storage		P_k	m	θ	C	C_F	C_{th}

b)

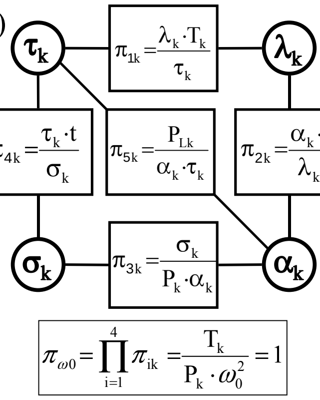


Figure 3. a) Physical variables describing energy transformation, b) relations of the variables in similarity theoretical graph representation according to [Macfarlane1964]

The intensity and quantity variables can be classified in flow and effort quantities. Figure 3 shows the physical variables describing the energy transformation (a) and their similarity relations in graph representation according to [Macfarlane1964] (b). Applying the Pi theorem [Buckingham1914] to the relations between the parameters, the ratios π_{ik} as shown in figure 3b result. By multiplication of these ratios along the outer edges of the graph, the eigenfrequencies of the lossless mechanical, electromagnetical and fluidic systems can be calculated without solving the corresponding differential equations (see equation in figure 3b). The reason for this is that the coupling of the ratios represents the entire energy conversion during the oscillation and thus the characteristic value of the oscillating system.

conducting energy	$E_{lk} = (\{x_l, x_m\} \subset X) \wedge (((x_l = x_m = \tau_k \vee \alpha_k) \wedge (\lvert\pi\rvert = \left\lvert\frac{x_l}{x_m}\right\rvert = 1)) \vee (x_l = \tau_k \wedge x_m = \alpha_k))$
storing energy	$E_{sk} = (\{x_l, x_m\} \subset X) \wedge (\{x_l, x_m\} = \{\alpha_k, P_k\} \vee \{\lambda_k, T_k\})$
converting energy	$E_w = E_{wk} \vee E_{wlm}$ $E_{wk} = (\{x_l, x_m\} \subset X) \wedge ((x_l = x_m = \tau_k \vee \alpha_k \vee \lambda_k) \wedge (\lvert\pi\rvert = \left\lvert\frac{x_l}{x_m}\right\rvert \neq 1))$ $E_{wlm} = (\{x_l, x_m\} \subset X) \wedge (x_l, x_m) \in X_l \times X_m \rvert_{l \neq m}$ $X_l = \{\tau_l, \alpha_l, \lambda_l\}; X_m = \{\tau_m, \alpha_m, \lambda_m\};$ $l, m \in k = \{1, 2, ..., 5\}$

Figure 4. Characteristic functions for the selection of solutions based on logical and set theoretical relations

The conduction of energy is decisively determined by the intensity variables, like force and electric current. The storage of energy is attached to a storage variable, e.g. the stiffness of a spring c in a mechanical system, and a quantity parameter, e.g. the displacement s. Converting energy comprises changing the absolute value of a physical quantitiy, e.g. during the intensification of a force, and the transformation of a quantity to a parameter of another physical partial system. An example for this kind of energy conversion is the lengthening of a rod by increasing the temperature (thermomechanical conversion). All intensity and quantity parameters can be converted in general. To reduce the complexity of the methodical procedure, the conversion of the quantity effort variable is attributed to the conversion of the intensity flow variable by division with the flow storage variable P.
It is necessary for the selection of solutions that the set $X = \{x_1, x_2, ..., x_n\}$ of the relevant physical parameters are known which describe a partial solution. A solution is suitable to conduct, store or convert energy if at least two quantities, indicated as x_l and x_m in general, are contained in the set X of the relevant parameters and fulfil the corresponding characteristic logical functions in figure 4.
Figure 5 presents an exemplary application of the approach to a given, solution describing set of π-ratios. It is shown that an electromagnet and the piezo effect are suitable for an electromagnetic energy conversion on principle.

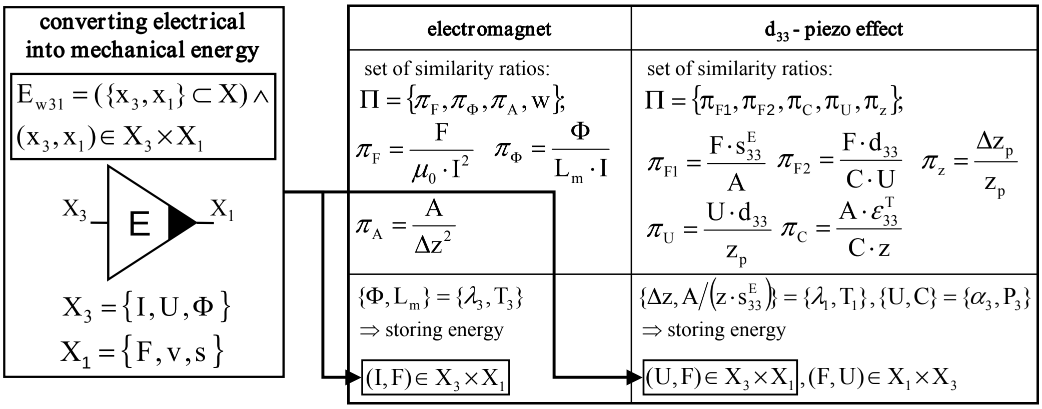


Figure 5. Systematical selection of solutions for electromagnetic energy conversion

The most important advantage of the discussed approach is that the scope of design solutions can be enlarged in comparison with other approaches. Additionally, the method can applied to identify easily the potential of functional integration of solutions.

2.2 Elaboration of principle solution variants by combining and varying

Exemplarily, an electromagnet is chosen to realize the electromagnetic conversion of energy. For a compatible connection with other partial solutions, a first embodiment and rough configuration of the single function carriers have to be developed. To generate friction between the drive and output end of the clutch, the magnetic force has to affect a movable part, e.g. an armature, of the drive or output side. For example, the electromagnet is positioned between the drive and output side (see variant I in table 1).

As a simple design for an armature, a single disc is defined which moves axially from the output end to the drive end and brings both sides into contact. The magnetic force generates Coulomb friction between armature disc and drive end. In connection with a lever arm, the friction generates the clutch torque. The conducted electrical energy does not branch out until synchronization, so that the sub-functions 1 to 3 are connected in series (see figure 2a). For a compatible serial combination of partial solutions, it is necessary that a function quantity created by a solution as output parameter matches with one input parameter of a solution which is ordered subsequently concerning to the functional interrelation. A characteristic ratio equation describing the transfer behaviour of the coupled solutions can be derived algorithmically by multiplication of π-ratios on the left and on the right of the equals sign of single ratio equations. The prerequisition for a compatible combination is that the matching inputs and outputs eliminate each other. Selecting the Coulomb friction as partial solution to generate friction and the torque-force-relation of the mechanics for the conversion of the friction force into the clutch torque, the following aggregated ratio equation can be deduced:

$$\pi_{Fm} \cdot \pi_{Fr} \cdot \pi_M = \tfrac{1}{4} \cdot i \cdot w^2 \cdot \mu \cdot \pi_A \qquad (2)$$

$$\Leftrightarrow \left(\frac{F_m}{\mu_0 \cdot I^2}\right) \cdot \left(\frac{F_r}{F_m}\right) \cdot \left(\frac{M}{F_r \cdot d_m}\right) = \tfrac{1}{4} \cdot i \cdot w^2 \cdot \mu \cdot \left(\frac{A}{\Delta z^2}\right)$$

F_m: magnetic force, F_r: friction force, M: torque, I: electric current, μ_0: magnetic constant, d_m: middle friction diameter, w: number of windings, i: number of friction surfaces, A: from the magnetic flux flushed area, μ: coefficient of static friction, Δz: width of the air gap

For dimensioning the clutch depending on the time of synchronization and the parameters of the engine and the output machine, the transient slip during engagement has to be considered (q.v. both back couplings of the clutch torque in figure 2a). The quantitative description of the synchronization can be derived by the impulse momentum theorem (see figure 2b). As is well known, two coupled, dimensionless differential equations result:

$$\left(1 - \frac{M_1}{M_k} = -\frac{\theta_1 \cdot \dot{\omega}_1}{M_k}\right) \wedge \left(1 - \frac{M_2}{M_k} = \frac{\theta_2 \cdot \dot{\omega}_2}{M_k}\right) \qquad (3)$$

M_1: driving torque, M_2: engine torque, M_k: clutch torque,
θ_1, θ_2, ω_1, ω_2: mass moments of inertia and angular velocities of drive and output
It is important for dimensioning to consider the synchronization time, e.g to check the thermal stability. With $M_i \neq M_i(\omega)$, $\forall i = \{1,2\}$, $\Delta\omega = \omega_1(t = 0) - \omega_2(t = 0)$, $\Delta t = t_0 - t_s$ and $\omega_1 = \omega_2$ arises the following ratio based solution for the differential equations:

$$\pi_{Mk} - \pi_{M1} - \pi_{M2} = 1, \qquad (4)$$

$$\pi_{Mk} = \frac{M_k \cdot \Delta t}{\Delta\omega \cdot \theta^*},\ \pi_{M1} = \frac{M_1 \cdot \Delta t}{\Delta\omega \cdot \theta_1},\ \pi_{M2} = \frac{M_2 \cdot \Delta t}{\Delta\omega \cdot \theta_2},\ \theta^* = \frac{\theta_1 \cdot \theta_2}{\theta_1 + \theta_2}$$

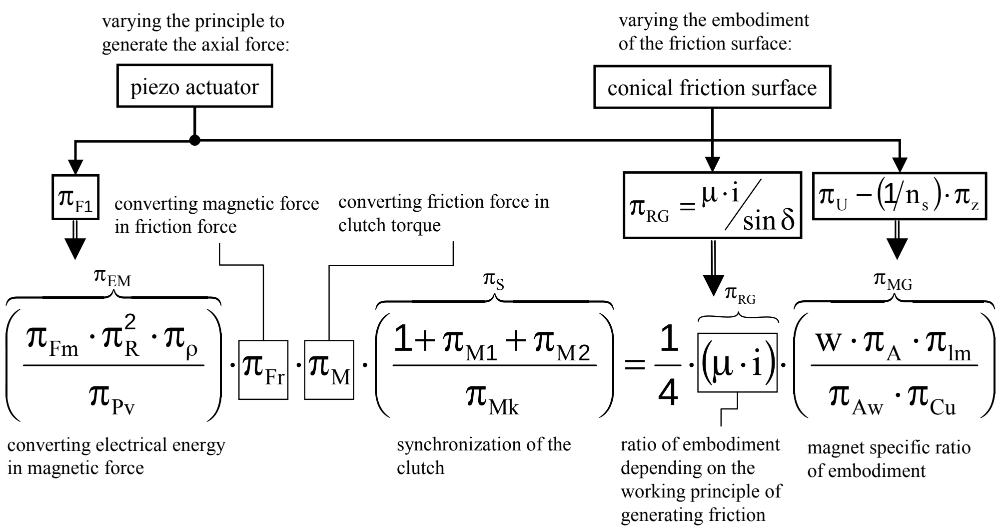
The addition of the nondimensional ratios in equation 4 characterizes a functional parallel connection of the torques M_1, M_2 and M_k. The negative algebraic signs of π_{M1} and π_{M2} points out that driving and engine torque work against the clutch torque and that there exists a degenerative feedback. Already in [Franke1976] and [Franke2005] was shown that essential over-all properties of a technical system can be derived by the topology of coupling. The ratio equation for π_{Mk} is linked directly with equation 2 by elimination of M:

$$\frac{\pi_{Fm} \cdot \pi_{Fr} \cdot \pi_M}{\pi_{Mk}} = \frac{i \cdot \mu \cdot w^2 \cdot \pi_A}{4 \cdot (1 + \pi_{M1} + \pi_{M2})} \qquad (5)$$

To get a simple manageable relation for approximate layout design, equation (5) is augmented by multiplication with further dimensionless characteristics. The add-on ratios enable the elimination of undesired function quantities und the replacement by variables on which requirements and known constraints can be referred directly.

Besides, design quantities can be concretized as far as that they can be represented by geometry variables which enable a rough estimation of package dimensions. In general, the voltage size controls the magnetic force during the operation of the clutch. For this reason, the electric current I is replaced by the voltage U in π_{Fm} according to Ohm's law. Specific material and geometry parameters determine the ohmic resistance of the windings. The ohmic resistance generates thermal power loss which limits the size of the allowed voltage.

The number of windings and the wire diameter are substituted by the copper fill factor and the winding area [Kallenbach2003]. Furthermore, the heat transfer area, the difference of temperatures between the coil inside and the environment and the coefficient of heat transfer determine the power loss. A simplifying assumption is that the area flushed from the magnetic flux has the shape of a circular ring and is identical to the heat transfer area. The ratio relation for the description of the electromagnetically switched disc clutch (see variant I in table 1) is shown in figure 6.



$$\left(\frac{\pi_{Fm}\cdot\pi_R^2\cdot\pi_\rho}{\pi_{Pv}}\right)\cdot\pi_{Fr}\cdot\pi_M\cdot\left(\frac{1+\pi_{M1}+\pi_{M2}}{\pi_{Mk}}\right)=\frac{1}{4}\cdot(\mu\cdot i)\cdot\left(\frac{w\cdot\pi_A\cdot\pi_{lm}}{\pi_{Aw}\cdot\pi_{Cu}}\right)$$

$$\pi_{RG}=\mu\cdot i/\sin\delta \qquad \pi_U-(1/n_s)\cdot\pi_z$$

$$\pi_{Pv}=\frac{R\cdot A\cdot\alpha_K\cdot\Delta\vartheta}{U^2}=1;\quad \pi_\rho=\frac{\rho_{el}\cdot l_m}{R\cdot A_{Dr}}=\frac{1}{w};\quad \pi_{Cu}=\frac{A_{Dr}\cdot w}{A_w\cdot k_{Cu}}=1;\quad \pi_R=\frac{R\cdot I}{U}=1;\quad \pi_{A_w}=\frac{A_w}{b\cdot B}=1;\quad \pi_{l_m}=\frac{l_m}{\pi\cdot d_m}=1$$

R: ohmic resistance, A: heat transfer area, α_K: coefficient of heat transfer, $\Delta\vartheta$: difference of temperatures between the coil inside and the environment, U: voltage, ρ_{el}: specific electrical resistance of windings,w: number of windings, A_{Dr}: wire diameter, A_W: winding area, k_{Cu}: copper fill factor, I: electric current, b: length of magnet, B: width of magnet, l_m: true length of windings, d_m: middle friction diameter, i: number of friction surfaces, μ: coefficient of static friction

Figure 6. Aggregated ratio equation of an electromagnetically switched friction clutch and ratio based variations of the principle solution

The equation in figure 6 presents couplings of ratios that are characteristic for the solutions. The ratios can be applied e.g. as "classifying criteria" and "solution characteristics" of a design catalogue. The values of the ratios can be stored as compressed solution knowledge in a design catalogue and used in following design processes. The so far usual, but unsatisfying designation in solution catalogues, e.g. in the "force-generator"-catalogue [Franke1972], like "large" and "small", can be quantified in that way.

The aggregated relation of ratios in π_{EM} summarizes the single ratios which are characteristic for the electromagnetic energy conversion of an electromagnet. For instance, it is readily identifiable that an electromagnet produces always thermal power loss which has to be considered by the embodiment of the clutch, e.g. by a sufficient heat transfer. The characteristic π_{MG} consists of magnet specific geometry ratios the structure of which depends on the shape and configuration of the magnet in the system. π_{Fr} and π_{RG} are typical for the fact that solid state friction is used as working principle for generating friction. π_M is the necessary ratio to represent the torque conversion. The coupled characteristic π_s describes the synchronization of the clutch. Invariant ratios and proposals for variations can be derived systematically from the equation in figure 6. The quantities π_M and π_s are invariants of a friction clutch. The other ratios can be changed for a definite task by variation of the working principle or shape.

If the magnetorheological effect is used for switching the clutch for example, the Newton fluid friction always works as friction effect. In this case, the characteristics π_{EM}, π_{Fr}, π_{RG} and π_{MG} can be replaced easily by the corresponding ratios of the magnetorheological solution. For a fixed working principle, like the Coulomb friction force, different variations of embodiment can be taken into account. For instance, the number of friction surfaces i can be varied or the coefficient of friction μ can be substituted by the corresponding parameter μ/sin δ of a conical friction surface (see figure 6). For this purpose, efficient algorithms can be defined with tools, like Maple or Mathematica.

2.3 Comparative dimensioning und basic optimization

By summarizing and transforming the terms in the equation of figure 6, a ratio relation arises to calculate the main dimensions of the clutch. This equation contains all important parameters and is very appropriate for layout design. In this case, the design parameters are combined to three dimensionless groups the dependencies of which can be demonstrated concisely in two dimensions. The description is highly operational if every variable design parameter appears only in one ratio number. Figure 7 shows the characteristic equation with the relevant parameters and the existent restrictions, e.g. relating to material and package dimension, according to the size and the direction of optimization of the design parameters.

$$B < \tfrac{1}{2}(D-d_i) \qquad < 0{,}7 \qquad d_i + B < d_m < D - B \qquad \alpha_K \approx 6{,}5\,\tfrac{W}{Km^2} + 0{,}05\Delta\vartheta\,\tfrac{W}{K^2m^2}$$

task identification π_{Mk}

$$\underbrace{1+\frac{\Delta t}{\Delta\omega}\left(\frac{M_1}{\theta_1}+\frac{M_2}{\theta_2}\right)}_{\pi_{Mk}} = \left(\pi/4\right)\cdot i\cdot \underbrace{\boxed{\mu}\cdot\left(\frac{\boxed{B^3}\cdot b}{\Delta z^2}\right)\cdot\left(\frac{\boxed{k_{Cu}}\cdot\mu_0}{\boxed{\rho_{el}}\cdot\Delta t}\right)}_{\pi^*_{em}}\cdot\underbrace{\left(\frac{\boxed{d_m^2}\cdot\Delta t^2}{\Delta\omega\cdot\theta^*}\right)\cdot\boxed{\Delta\vartheta}\cdot\boxed{\alpha_K}}_{\pi_{tm}}$$

$$\mu \in \{0{,}1;\ldots;0{,}6\} \qquad \rho_{el} \in \{0{,}015;\ldots;0{,}07\}\,\Omega\cdot mm^2/m \qquad \Delta\vartheta < \Delta\vartheta_{zul} \in \{50;\ldots;240\}K$$

directions of optimization to generate a large clutch torque:

- high values for: middle diameter of friction d_m, width of magnet B, length of magnet b, number of friction surfaces i, copper fill factor k_{Cu}, allowed difference of temperatures Δϑ, coefficient of static friction μ
- small values for: specific electrical resistance of windings ρ_{el}, width of air gap Δz

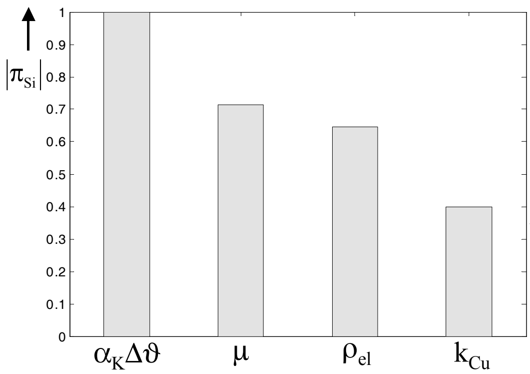
Figure 7. Dimensioning equation of an electromagnetically switched friction clutch

The left side of the equation in figure 7 is to be interpreted as a nondimensional task identification [Franke1999] which is a function of the quantities of the engine and the output machine and the time of synchronization. This ratio is completely determined by the requirements (see the specifications of the case study stated below) and so it is a characteristic invariant of the design task.

For a given task, the task identification can be derived by dimensional analysis, e.g. on basis of the Pi theorem. If solution descriptions in form of Pi terms exist, like in figure 7, suitable solutions can be chosen directly with help of the dimensionless task identification. The right side of the equation is determined by the product of the nondimensional groups π_{em}* and π_{tm}. π_{em}* is an electromechanical ratio which contains mechanical parameters, like the coefficient of friction, and electromagnetic quantities, e.g. the specific electrical resistance of the coil. π_{tm} is a thermomechanical ratio which considers the middle friction diameter and the coefficient of heat transfer among other things. The equation in figure 7 clarifies e.g. that the radial magnet dimension B is cubed and therefore more sensitive in calculation than the middle friction diameter d_m and the axial length b. To get a large torque moment, the width of magnet B should be preferably as large as possible.

Additionally, sensitivity analyses are useful to find out systematically the greatest effects of variations depending on the restrictions. Figure 8 shows the results of a local sensitivity analysis which calculates the effects of changing material parameters relating to a basic variant. As basic values for the dimensions of the magnet, the values determined by the case study stated below are used. The nominal values of the analyzed parameters are the medians of the solution intervals (see figure 7). Figure 8 shows graphically the absolute values of dimensionless sensitivity coefficients π_{Si} which can be calculated with the aid of partial derivatives of the task identification and the standard deviations of the quantities.

The standard deviation is the difference between the median and the boundary value of the solution interval. The largest coefficient is standardized to the value „1“. The variation of the variable $\alpha_K\Delta\vartheta$ has the largest effect and a modification of the copper fill factor k_{Cu} the smallest effect on task identification π_{Mk}. To increase the task identification, the coefficient of heat transfer should be improved and a magnet with a higher allowed excess temperature should be used particularly. An estimation of errors and negative effects could be described with ratio representation as well [Germer2005].



dimensionless coefficients of sensitivity

$$\pi_{Si} = \left(\frac{\partial \pi_{Mk}\cdot\sigma_{xi}}{\partial x_i}\right)\cdot\left(\max\left(\frac{\partial \pi_{Mk}\cdot\sigma_{xi}}{\partial x_i}\right)\right)^{-1}$$

dimensionless vector of sensitivity

$$\overline{\pi}_S = \begin{pmatrix}\pi_{S\alpha\Delta\vartheta}\\ \pi_{S\mu}\\ \pi_{S\rho el}\\ \pi_{SCu}\end{pmatrix} = \pi_{\alpha\vartheta}\begin{pmatrix}\pi_{\alpha\vartheta}^{-1}\\ \sigma_\mu/\mu_0\\ \sigma_{\rho el}/\rho_{el0}\\ \sigma_{kCu}/k_{Cu0}\end{pmatrix}, \pi_{\alpha\vartheta} = \frac{(\alpha_K\Delta\vartheta)_0}{\left(\frac{\partial(\alpha_K\Delta\vartheta)}{\partial\Delta\vartheta}\right)\cdot\sigma_{\alpha\Delta\vartheta}}$$

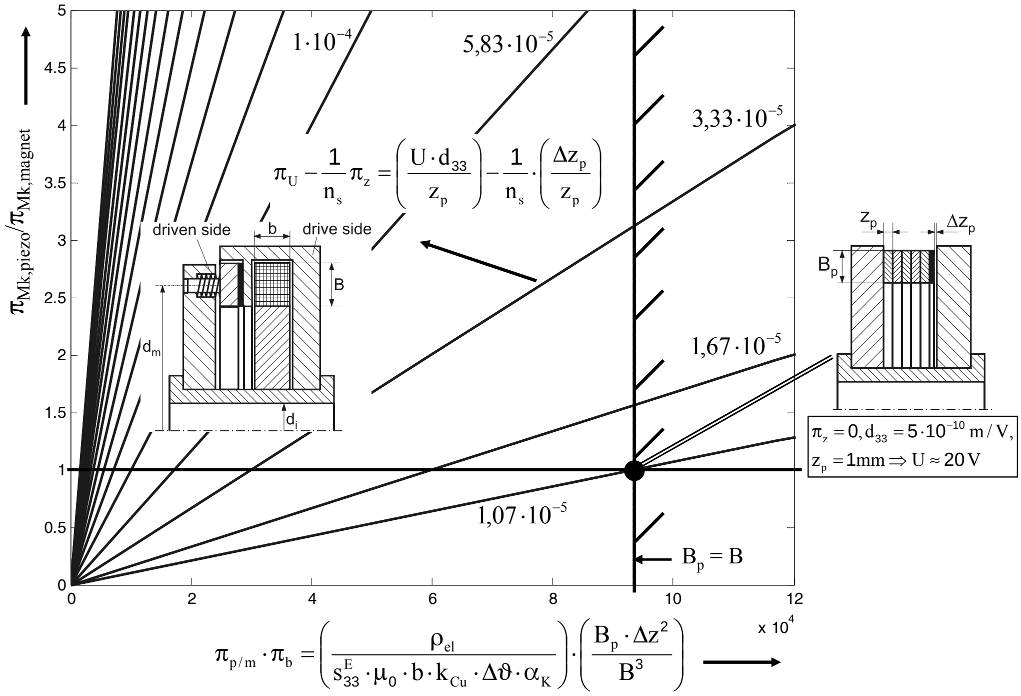
$\sigma_i = \Delta x$: standard deviation

x_0: nominal value of physical quantity

Figure 8. Local sensitivity analysis based on nondimensionless ratios

The electromagnetic disc clutch is to be compared with a piezoelectric clutch (variant IV in table 1) first. The ratio equation of the piezoelectric clutch can be derived from the magnetic clutch equations by substituting the magnet by a piezoceramic actuator which generates the axial force. In the characteristic equation of the clutch (q.v. figure 6), the ratio π_F (see figure 5) replaces π_{EM} and (π_U – $(1/n_s)\pi_z$) substitutes $\pi_{MG.}$

The configuration of the actuator in the clutch and the corresponding equation are shown in table 1. The fundamental differences between a magnetic and piezoelectric clutch are: the piezo actuator works without important power loss during the quasi static operation in contrast to the magnet (no loss-ratio π_{Pv} is existent). The magnet width B is much more sensitive than the width of the piezo actuator B_p.



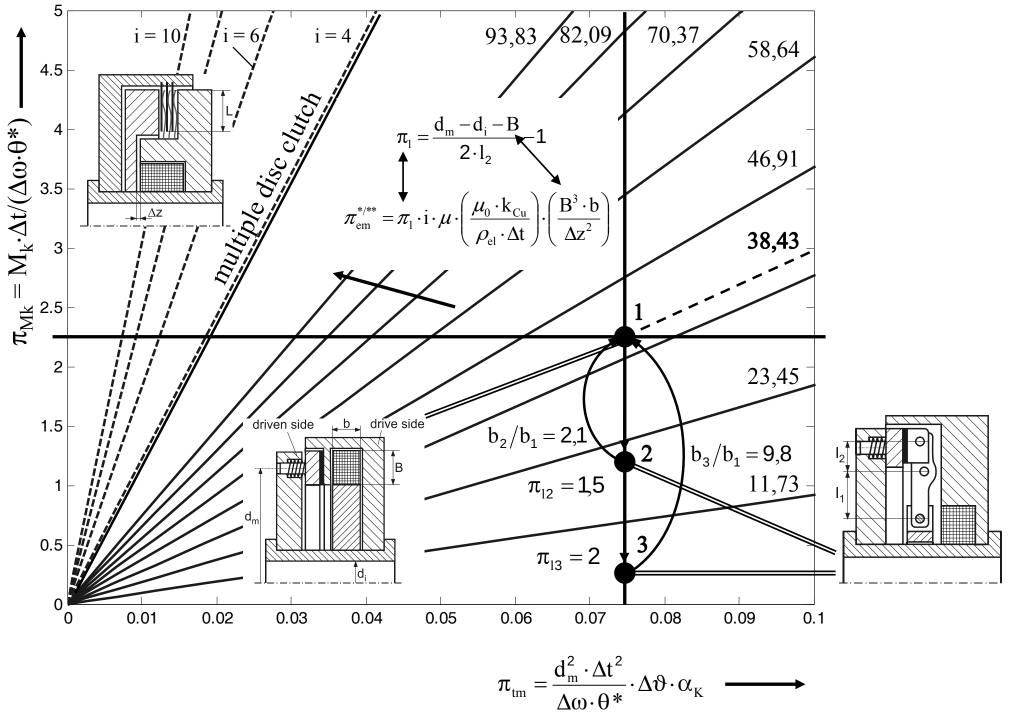
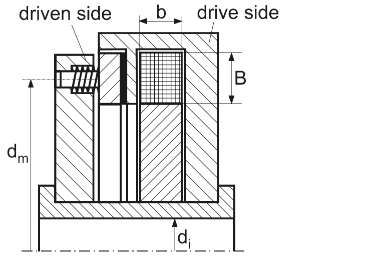
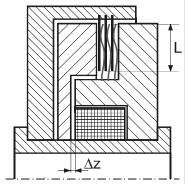
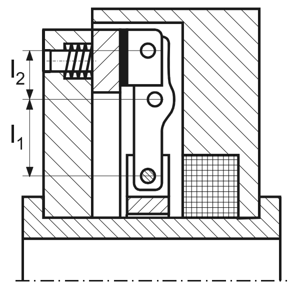
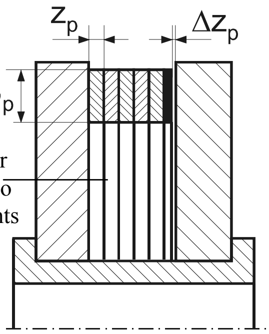


Figure 9. Evaluation, dimensioning and optimization with dimensionless sets of characteristic curves, a) comparison between magnetic and piezoelectric clutch, b) dimensioning of a magnetic clutch

Table 1. Clutch variants and the corresponding ratio equations

operations of variation ↓	principle sketch	characterictic π-ratio equations $^{1)}L \geq 2 \cdot M_k / (i \cdot \mu \cdot d_m^2 \cdot p_{zul})$
increasing the number of friction surfaces	I)  II) 	$\pi_{Mk} = \left(\pi/4\right) \cdot i \cdot \underbrace{\mu \cdot \pi_{em}}_{\pi^{*}_{em}} \cdot \pi_{tm}$ $\pi_{em} = \frac{\mu_0 \cdot k_{Cu}}{\rho_{el} \cdot \Delta t} \cdot \frac{B^3 \cdot b}{\Delta z^2}$ $\quad \pi_{tm} = \frac{d_m^2 \cdot \Delta t^2}{\Delta\omega \cdot \theta *} \cdot \Delta\vartheta \cdot \alpha_K$ I) i =1: disc clutch: $\pi_{Mk}/\pi_V = 1$ $1 + B/d_i \approx d_m/d_i \leq (D - B)/d_i$ II) i >1: multiple disc clutch: $\pi_{Mk}/\pi_V \approx i$ $1 + 2B/d_i \approx d_m/d_i \leq (D - L^{1)})/d_i$
intensifying the axial force	III) 	$\pi_{Mk} = \left(\pi/4\right) \cdot i \cdot \underbrace{\mu \cdot \pi_l \cdot \pi_{em}}_{\pi^{**}_{em}} \cdot \pi_{tm}$ $\quad \pi_l = l_1/l_2$ $1 + B/d_i + 2\pi_l \cdot (l_2/d_i) \approx d_m/d_i \leq (D - L^{1)})/d_i$ $\pi_l = 1{,}5 \Rightarrow \pi_{Mk}/\pi_V \approx 0{,}6$ $\pi_l = 2 \Rightarrow \pi_{Mk}/\pi_V \approx 0{,}25$
varying the principle of generating the axial force	IV) 	$\pi_{Mk} = \left(\pi/2\right) \cdot i \cdot \mu \cdot \pi_m \cdot \left(\pi_U - \frac{1}{n_s} \cdot \pi_z\right)$ $\pi_m = \frac{\Delta t}{\Delta\omega \cdot \theta *} \cdot \frac{B_p \cdot d_m^2}{s_{33}^E}$ $\quad \pi_z = \frac{\Delta z_p}{z_p}$ $1 + B_p/d_i \approx d_m/d_i \leq (D - B_p)/d_i$ $\quad \pi_U = \frac{U \cdot d_{33}}{z_p}$ $\Delta z_p = 0 \Rightarrow \pi_{Mk}/\pi_V \approx 30$ $\Delta z_p = 0{,}01mm \Rightarrow \pi_{Mk}/\pi_V \approx 0{,}03$

For a better comparability of piezoelectric and magnetic clutch, the quotient of both relevant ratio equations has been evaluated. Figure 9 presents dimensionless sets of characteristic curves for evaluation, dimensioning and optimization of a clutch.

The dot in figure 9a represents a piezoelectric clutch which fulfils the quantitative values of the task in the case study ($\pi_{Mk,piezo}/\pi_{Mk,magnet} = 1$). Calculations with the dimensionless groups show clearly that in the case of a constant actuator width ($B_p = B$), constant friction diameter, a piezoceramic thickness of 1 mm and a nonexistent air gap ($\pi_z = 0$), a voltage of approximately 20 V is sufficient to generate the necessary clutch torque. In this example, the piezo solution IV creates a 30 times larger torque per design space (π_{Mk}/π_V) than the magnetic variant I. Increasing the width of the air gap to only 0,01 mm by keeping π_U constant, the torque per design space (π_{Mk}/π_V) decreases to the thousandth part. It would be nearly perfect concerning design engineering, to design a shifting clutch without air gap. To avoid immoderate abrasion, the drive and driven side are to be separated completely after shifting. For this reason, the piezoelectric effect is less suitable than other working principles for actuating a friction clutch in general.

Alternatively, a multiple disc clutch and a disc clutch with force intensification are analyzed concerning the ability to fulfil the given specifications. The multiple disc clutch (variant II in table 1) can be described by the same dimensionless equation as the single disc clutch. The difference to the single disc clutch is that the number of friction surfaces i increases and therefore the value of the ratio π_{em}* (see table 1) is higher keeping the other parameters constant. Taking dot 1 in figure 9a as starting position, the dashed curves show the behaviour of the multiple disc clutch (variant II).

The force intensification of variant III in table 1 is realized mechanically by a lever with the increasing factor π_l. Considering the functional point of view, the force intensification corresponds to the integration of a partial function into the serial functional interrelation of generating friction force (see figure 2a). The ratio relation π_{Mk} can be derived by the corresponding equation of the single disc clutch by multiplication of π_l. Increasing the factor π_l keeping the middle friction diameter d_m and length b constant, the width B and the clutch torque M_k decreases because of the restrictions of package dimensions (q.v. dots 2 and 3 in figure 9b). To enhance π_{Mk} to the value of the starting point 1, the axial length b has to be increased by factor 2, if $\pi_l = 1,5$, and by factor 10, if $\pi_l = 2$. The result of increasing b is the decrement of the torque per design space in comparison with variant I. The calculation emphasizes that the multiple disc clutch is able to generate the largest torque per design space compared to the other magnetic clutch variants.

Table 1 presents the designed clutches, the corresponding ratio equations and the torque per design space (π_{Mk}/π_V) of each solution. The springs of variants I and III for the retraction of the armature disc (q.v. in table 1) are neglected simplifying the establishment of the nondimensional characteristics. It has to be considered for final embodiment that the allowed values for surface pressure and friction area temperature are complied with. In the following, a possible methodical procedure is pointed out for the size selection of a magnetic clutch.

Table 2. Relevant requirements for the design of a shifting clutch

requirements	values
driving torque / engine torque	$M_1 = 50$ Nm / $M_2 = 25$ Nm
mass moments of inertia of drive / engine	$\theta_1 = 0,04$ kgm^2 / $\theta_2 = 0,02$ kgm^2
differences of angular velocities at t = 0	$\Delta\omega = 400$ s^{-1}
time of synchronization	$\Delta t < 0,2$ s
inside diameter	$d_i = 25$ mm
outside diameter	D < 120 mm

Table 2 lists the relevant quantifiable requirements which shifting clutches have to fulfil. With these specifications, as a first step the value of the task identification can be calculated to $\pi_{Mk} = 2,25$. Presuming a maximum middle friction diameter of approximately 100 mm and typical values for the allowed excess temperature and coefficient of heat transfer, the value of the characteristic π_{tm} can be determined ($\pi_{tm} \approx 0,074$). With the parameters π_{Mk} and π_{tm}, the value of the dimensionless group π_{em}* can be calculated to π_{em}*≈ 38,4 (see dot 1 in figure 9b). To generate the clutch torque in a minimal design space, B is to be maximized as geometry parameter of highest sensitivity considering the design space restrictions. Taking typical values for copper fill factor, air gap, coefficient of friction and the specific resistance of the coil, the width B and length b can be estimated. With $k_{Cu} = 0,7$, $\rho_{el} = 1,5\cdot10^{-8}$ Ωm, $\mu = 0,4$ and $\Delta z = 1$ mm succeeds: B = 32 mm, b = 10 mm. The particular advantage of the ratio representation is that not only influences of geometry variations, but also the effects of changes concerning requirements and mounting conditions can be identified and used systematically for dimensioning. For example, the same clutch torque could be generated if magnet length b increases by factor k and the friction diameter decreases by factor $1/(k)^{1/2}$.

3. Conclusions

The paper shows that similarity ratios support holistically the methodical synthesis of solutions. Partial solutions can be described and selected on the basis of dimensionless characteristics. The usage of ratios simplifies the representation of solutions because of the reduction of parameters. The characteristics used can be aggregated to complete ratio relations for the representation of principle solutions according to a required functional interrelationship. The aggregated characteristics are mathematically formalized and thus compressed solution knowledge which is appropriate for dimensioning and comparative calculations. The coupled ratios can be used as fixed solution principles for subsequent design processes, e.g. in design catalogues. The configuration of the dimensionless groups reveals the structural properties of solutions, e.g. relevant design parameters and used principles of coupling, like parallel or serial connections. These properties enable simplified approaches for systematic variations and optimizations of solutions.

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